nature portfolio

Peer Review File

Superluminal Correlations in Ensembles of Optical Phase Singularities

Corresponding Author: Professor Ido Kaminer

This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Attachments originally included by the reviewers as part of their assessment can be found at the end of this file.

Version 0:

Reviewer comments:

Referee #1

(Remarks to the Author)
I have an overall good opinion of the manuscript and I think it should be published after a few clarifications.

See the attached detailed review.

Referee #2

(Remarks to the Author)
This work provides an experimental measurement of phase singularities in random scalar waves, verifying statistical properties like pair correlation, velocity probability densities, and distance-velocity correlation, explaining how nearby singularities of opposite charge can move faster than light. I have never seen real optical phase singularities drifting around over time and crashing into each other, so the supplementary video alone was very enjoyable to watch. Although I do not have strong experimental expertise, the authors provide an extensive account of their methodology across the main text and supplementary information and I found their presentation of results convincing.

The main strength of the work is the observation of singularities moving faster than light when created and annihilated in pairs and measurement of velocity and distance velocity probability distributions. The superluminal speed of phase singularities has been understood beyond much doubt in theory for a fairly long time (e.g., Ref. 8) so the presented results aren't surprising per se - but this does not diminish importance. Experimental verification also carries significant value as a technical achievement, in terms of the high simultaneous spatial and temporal resolution and reconstruction of the time-varying field. It ties in well with the current popularity of time-varying metaoptics and it is certainly of interest to researchers in optics. I believe it could have a significant impact simply as a demonstration of a way to observe the time-dynamics of delicate sub-wavelength features of light, as the authors conclude.

As the authors mention, phase singularities are a universal feature of structured waves and have a broad appeal beyond optics. I can also see the work's headline result (and supplementary video) being of interest to the general public. On this basis I would recommend publishing in Nature. However, I would like the authors to address my queries below:

1. On line 50: The authors point out phase singularities “serve as carries of classical and quantum information”. But later on line 158: “Phase singularities carry zero intensity and thus can move superluminally”. The authors should reconcile these two statements a bit more carefully, e.g., explaining where the information is carried (i.e. not in the singularity core). Given this the authors should justify the statement of “suggesting phenomena of ultrafast information flow” in the abstract.

2. In the section starting on line 87: I understand the suitability of the experimental setup appropriate for making singularities observable, but why is it appropriate for generating random scalar waves in the first place? What produces the Gaussian randomness in the distribution of the phase singularities? Does it come from the hBN structure? Likewise, how do the time

dynamics of the singularities relate to the excitation? This would provide useful context for someone minded more towards pure singular optics theory.

3. On line 200: The authors say the correlation function g(R) is “defined as the probability of finding a pair of singularities at a distance R from one another” This seems imprecise because g(R) exceeds 1 - is it not a measure of the local density of singularities, rather than a probability?. This also occurs on line 397 and in section VI of the supplementary information.

4. In the text around Figure 3 (and the figure’s caption), descriptions for Fig. 3(d) and (e) seem to be the wrong way round.

5. In Figure 4 - showing numerical colourbar limits would be useful.

6. In the conclusion the authors suggest different platforms might provide a way to measure different kinds of singularity. Is there any realistic prospect for measuring the time-dynamics of vector (polarisation) singularities or larger scale topologies (like near-field optical skyrmions). Or is this what is meant by “multi-dimensional” singularities?

Referee #3

(Remarks to the Author)

The paper is devoted to tracking the dynamics of phase singularities near annihilation events, with the detection of short-term superluminal motion. An assessment of both individual annihilation events and a statistical analysis in phase space is proposed. And it is shown that the measured distance correlation agrees with the particle-like nature of singularities, and the distribution of velocities violates the corpuscular analogy. An experimental solution for assessing the dynamics of singularities and their statistical properties is proposed. The results are original and interesting.

The article lacks attention to detail, which affects the overall understanding of the problem and leads to some uncertainty in the processing of the results. The conclusions are also not reliable enough. There are sufficient references to previous works.

But there are comments and remarks

-A physical approach to studying phase singularities is the interference of Gaussian random two-dimensional beams. What happens in the three-dimensional case if we are talking about the near field?
-The conclusion is somewhat unusual and seems naive that the observation of the maximum velocity is “limited not by the field properties, but by the state-of-the art of the microscopy”. Although at the moment the advanced microscopy, subwavelength focusing, smaller than the diffraction limit, is manifested even in the formation of photonic jets and photonic zigzags.

1. It is necessary to expand the paper with a deep physical interpretation of the obtained results with the velocity greater than the light velocity.
2. Phase retrieval has been implemented with the generalization of FERI theory. The explanation provided in the text is not sufficient to represent the generalization mechanism.
3. How is longitudinal (transversal) coherence taken into account under all these FERI transformations? It is absolutely unclear the influence of polarization of incident beam. How is controlled the intrinsic properties of incident beam, temperature changes in the sample under beam action and heating?
The term “spatial correlation” was introduced, although its application to singularity correlation and annihilation was conceptually flawed. The term “dynamical statistical properties of singularity pairs” is also introduced poorly.
The paper requires significant revision for possible publication

Referee #4

(Remarks to the Author)
I co-reviewed this manuscript with one of the reviewers who provided the listed reports.

Version 1:

Reviewer comments:

Referee #1

(Remarks to the Author)
I thank the authors for carefully addressing my comments. I suggest that the paper is accepted in the present form.

Referee #2

(Remarks to the Author)

The authors have answered my queries and made suitable revisions to the manuscript and supplementary material. On my side I am satisfied and can recommend publication.

Referee #3

(Remarks to the Author)
A. Summary of the key results – the changes were made
B. Data & methodology: validity of approach, quality of data, quality of presentation – the changes were made
C. Conclusions: robustness, validity, reliability – the changes were made
D. Suggested improvements: experiments, data for possible revision – the changes were made
E. Clarity and context: lucidity of abstract/summary, appropriateness of abstract, introduction and conclusions – the changes were made

After significant reworking, the paper can be published.

Referee #4

(Remarks to the Author)
I co-reviewed this manuscript with one of the reviewers who provided the listed reports.

We thank all the referees for their valuable feedback. We include a point-by-point response to address all the comments below. The referees' comments are in '**black**', our responses are in '**dark teal**', and changes made in the manuscript are marked in '**orange**'.

Referee #1:

I have an overall good opinion of the manuscript and I think it should be published after a few clarifications. See the attached detailed review.

We thank the referee for supporting the publication of our manuscript and for the thoughtful and interesting comments. We were happy to include them in the revised manuscript.

General Assessment

The manuscript reports ultrafast imaging of optical phase singularities in hexagonal boron nitride (hBN) using a free-electron interferometric ultrafast transmission electron microscope (UTEM). The authors claim to directly observe "superluminal" velocities of phase singularities and to measure full phase-space correlations (distance–velocity joint distributions) in a singularity ensemble. The experimental resolution (20 nm spatial, 3 fs temporal) is impressive, and the technical implementation of phase-resolved UTEM represents a significant advance.

However, the central physical claim of superluminal correlations is conceptually misleading. The reported velocities correspond to the apparent motion of phase zeros, not to any transport of energy or information (as correctly pointed out in the manuscript).

We thank the referee for bringing this point up. We of course agree that transport of energy or information does not happen faster than light, and we made it clear in the original manuscript (as the referee also pointed out). In the revised manuscript, we further clarify this point and make a change to the terminology to help avoid any confusion from the start. See below an extended discussion about the choice of terminology and specifically regarding the word "superluminal".

This makes the term "superluminal" in the title inappropriate and indeed misleading in many ways that could be detrimental to the otherwise serious approach taken in the manuscript. The results instead demonstrate ultrafast correlation mapping of topological defects, an important and rigorous result on its own. I find that this work stands as an elegant and rigorous confirmation of long-predicted heavy-tailed statistics in singular optics, rather than a claim of superluminal phenomena. Also, the authors correctly present their findings as an experimental confirmation of a long-standing prediction, not as the discovery of a new phenomenon.

We completely agree. The first experimental observation of this old prediction is a central result of our work. In fact, the prediction of heavy-tailed statistics was what led to the expectation of extreme velocities and also motivated other studies of statistical properties and pair correlations. In the revised manuscript, we now make sure to give credit to the prediction of heavy-tailed statistics and emphasize its first observation as a central part of this work (added twice in the introduction and once in the results section under Eq. 4).

We address the "superluminal" terminology in the response below.

Major Comments

The main claims of this paper revolve around "superluminal correlations" and the observation of singularities moving faster than light. While the experimental methodology is excellent, the interpretation of these findings is problematic, largely due to a misleading use of the term "superluminal" and an unclear notion of what the authors mean by "correlation." Below, I expand on these conceptual issues and discuss why the title and framing of the work should be revised. To clarify these points, I also include in the Appendix several analytical examples and simple models that reproduce such apparent effects within fully causal physics. I used them to clarify to myself how these superluminal zeros might arise. These examples help make sense of the data and confirm that the reported heavy tails in the velocity distribution are both expected and interesting, but not acausal.

Misleading Claim of "Superluminal" Correlations

The manuscript repeatedly refers to "velocities exceeding the speed of light" (see lines 30–33, 157–159) and describes these as "superluminal correlations." However, the text itself acknowledges that phase singularities carry zero intensity and therefore can "move superluminally without energy or information transmission." This is indeed correct: the measured quantity is a geometric velocity of the phase field, not a physical or causal propagation speed. What is observed is the motion of field zeros in a complex interference pattern, a purely kinematic property of the evolving phase landscape.

It is therefore not clear what "correlations" are being described as superluminal. If this term is meant to denote a correlation function of singularity positions or velocities, then the adjective "superluminal" is misleading and unnecessary. No causation propagates faster than light; what is shown is that apparent velocities in the phase-space statistics can exceed *c*, which is a known property of random-wave fields.

For this reason, the current title exaggerates the phenomenon and risks confusion with genuine superluminal signal transport. A more appropriate framing would emphasize the remarkable experimental ability to resolve ultrafast correlations in ensembles of singularities, rather than to suggest a violation of causality. I would therefore recommend a title along the lines of:

"Ultrafast Correlations in Ensembles of Optical Phase Singularities."

We thank the referee for this detailed and thoughtful comment. We agree. The motion of singularities is a property of interference patterns – singularities do not transmit energy or information and can exceed the speed of light without breaking causality.

Thus, our work does not in any way violate causality. Rather, it demonstrates experimentally a property of the phase landscape that can move with arbitrary high velocities. The original submission did not mention causality because we took it for granted that it is preserved. We now note this explicitly in the revised manuscript.

The referee also generously acknowledged that the original manuscript did not make any claim of breaking causality. Since the terminology we used may have been confusing, we now revise the manuscript and use the term "apparent superluminal", as suggested by the referee, when the word superluminal appears for the first time. We also explicitly state that singularities exceed the speed of light without breaking causality.

We further use the term “apparent superluminal” throughout the entire manuscript. For example,

Intriguingly, these apparent superluminal velocities are paradoxically amplified by the slow group velocity of hyperbolic phonon polaritons in our material platform, hexagonal boron nitride membranes.

We also added an additional clarification to the nature of singularities:

… Most notably, theory has long predicted that optical singularities exhibit heavy-tailed statistics, containing extreme events of apparent superluminal motion (singularities exceed the speed of light without breaking causality). …

… This fact is a mathematical consequence of the continuity of the phase rather than a violation of physical laws: Phase singularities do not carry energy or information and thus can “move” superluminally without breaking causality[13]; their apparent superluminal motion is a pure kinematic property of the evolving phase landscape. …

We have no objection to altering the terminology further, if needed.

For context, there are two reasons we use the word “superluminal”, rather than alternative ones:

(1) The terminology of superluminal velocity is common in previous literature to describe motion of features like singularities that surpass the speed of light *without breaking causality* [Aharonov, *Phys. Rev.* **182**, 1400 (1969); Afshordi, *Phys. Rev. D* **75**, 083513 (2007); Bruneton, *Phys. Rev. D* **75**, 085013 (2007); Babichev, *JHEP* **02**, 101 (2008); Berry, *J. Phys. A: Math. Theor.* **45**, 185308 (2012); Jhajj, *Phys Rev X* **6**, 031037 (2016); Clerici, *Science Advances* **2**, e1501691 (2016); Gianfrate, *Light: Science & Applications* **7**, 17119 (2018); Dominici, *Phys. Rev. Research* **3**, 013007 (2021)].

(2) Before annihilation, the singularities follow trajectories that involve diverging velocities. So, any arbitrarily high velocity is in principle surpassed. However, c has a specific importance here. In the motion of singularities, it describes the scale of the heavy tail. Specifically (copying Eq. (4), based on Ref. [8], and substituting), the average singularity velocity is given by

$$\langle v \rangle = c\frac{\pi/\sqrt{2}}{\sqrt{1+(\lambda_0/\Delta\lambda)^2\left(v_\mathrm{g}/v_\mathrm{ph}\right)^2}}.$$

Thus, while the speed of light does not present a concrete threshold, it does determine the *scale* of relevant velocities for our experiment. A unique aspect of our experiment is the relatively wide bandwidth $\lambda_0/\Delta\lambda \sim 23 \pm 3$, and relatively slow group velocity $v_\mathrm{ph}/v_\mathrm{g} \approx 12 \pm 1$. Together, these create the nice coincidence that the average velocity is approximately c, rather than much lower velocities as in more common realizations of optical singularities. We now emphasize this motivation in the results section of the revised manuscript.

Regardless of these reasons, we would not insist on a particular choice of terminology. We are happy to consult with the editor about the wording in the title.

We also apricate the referee's extended notes about "Understanding the Result through Simple Models", providing further context for why the phase singularities can move faster than light. Following these notes, we added a new section to the Supplementary Information that presents a simple model and example for how singularities move faster than light. This section is copied at the bottom of the response to this referee.

Also, the abstract and conclusion (lines 40, 319–323) further suggest that these measurements point to "ultrafast information flow." This interpretation is unsupported by the data. No energy, entropy, or causal signal has been shown to propagate faster than c. The observed dynamics instead represent the topological rearrangement of zeros in a continuous optical field, a process that can exhibit formally unbounded apparent velocities without transmitting any information. This speculative link to "information flow" should therefore be removed, or at least substantially toned down.

Regarding lines 319-323, we do not suggest any faster-than-c information flow, but rather offer a connection, yet to be explored, between the ultrafast dynamics of singularity annihilation and superoscillations.

We of course agree that the information flow is subluminal. We modified the sentences accordingly.

In line 40:

Our findings deepen our understanding of phase-singularities and their universality, enabling to probe topological defect dynamics at previously unattainable timescales.

In lines 319–323:

Our experiment observes simultaneously, and distinguishes, the two phenomena: (1) superoscillations are inherently created by phase singularities as seen at every frame; (2) the apparent superluminal velocities are observed in the dynamics between frames. There seems to be an indirect, yet intriguing, connection between these distinct features of topological defects, calling for further research: The extreme dynamics of singularities appear to be a direct manifestation of superoscillatory field gradients, once undergoing temporal evolution.

Also, I have a doubt that I wish the authors to clarify. The authors report an average singularity velocity $\langle v \rangle \approx 1.04c$ (lines 240–255), obtained by relating the phase and group velocities of polaritons in hBN ($\frac{v_\phi}{v_g} \approx 12$). This number is not a physically meaningful speed; it arises from parameters of a random-wave model in which the normalization by the small group velocity artificially amplifies apparent phase gradients. Am I wrong? If not, this should be clarified.

The statement that slow group velocity “amplifies” superluminal motion misrepresents the mathematics; the effect is statistical, not dynamical. It reflects the well-known result that the conditional distribution of the zero-velocity $\dot{\mathbf{r}}_{\mathbf{0}} = -\partial_t \mathbf{A} \, / \, \nabla \mathbf{A}$ has heavy tails that can extend beyond c without any energy transport.

The presence of these heavy tails in the experimental velocity distribution (Fig. 3f) is in fact quite interesting. While one might initially suspect they arise from tracking noise or phase-unwrapping artifacts, theoretical models of random-wave ensembles predict precisely such non-Gaussian tails. Their experimental confirmation is therefore of genuine significance, as it validates a statistical feature that had remained difficult to observe directly.

We thank the referee for this important point, which we are happy to address in the revised manuscript. The reported average singularity velocity $\langle v \rangle$ is indeed a pure statistical quantity, obtained as the empirical mean of all measured singularity velocities in the experiment. From the theoretical side, according to random-wave theory, $\langle v \rangle$ increases with the variance of the wavevector distribution; in hBN, the extremely slow group velocity strongly increases this variance, thereby shifting more weight to higher velocities and yielding the observed large $\langle v \rangle \sim c$. We calculated that a similar experiment in free-space ($v_{\mathrm{g}} = v_{\mathrm{ph}}$) would have only 0.4% of the singularities at velocities about c (shown in SI Fig S4). Thus, the slow group velocity gives rise to a counterintuitive statistical enhancement of apparent superluminal speeds, without any dynamical amplification. We have revised the text to clarify this point and emphasize the purely statistical origin:

$\langle v \rangle$ is the average velocity of the singularities. This value is calculated by averaging all the measured velocities of singularities in the experiment (the distance that the singularity traveled between frames, divided by time step) and is measured directly in our experiment to be $\langle v \rangle \approx (1.04 \pm 0.04)c$.

This result also highlights a unique property of hBN PhPs: their slow group velocity enhances the variance of the wavevector distribution, thereby increasing the weight of extreme velocity singularities in the heavy tails of the distribution. This extended variance yields an average velocity that slightly exceeds the speed of light. In comparison, the average velocity would be much lower if not for the hyperbolic nature of hBN PhP.

Experimental Uncertainties

The automated tracking of approximately fifty singularities across 285 frames is technically impressive. However, the drift-correction scheme (Extended Data Fig. 2) relies on manually selected features and may not guarantee sub-pixel accuracy. Residual misalignments could produce small artificial apparent velocities. Because of this, it is essential to propagate uncertainty through to the velocity distribution and to include confidence intervals for the “superluminal” fraction. Without such an analysis, the statistical significance of the long-velocity tail cannot be properly assessed. I am not sure whether this was taken into account in the error.

This analysis of errors was performed. The majority of the analysis, including the identification of the singularities, is done automatically. The manual corrections are performed due to electron beam drifts. These drifts are inevitable due to the long-time acquisition in this experiment. This drift results in subtle translation movement between neighboring frames. We fix for translation using a conventional method in electron microscopy: identifying and following indicative features. With a typical number of 10 features, we get an over-imposed scheme (to determine two parameters per frame: drift in x, drift in y), therefore, the transformation calculation is robust. Specifically, the use of a K2 camera (1050 pixels on each axis, covering 20 nm per pixel) provides a substantial advantage and determines the resolution limit.

To summarize, the only manual part of the analysis was the feature selection used to correct for drifts. The statistical analysis of the singularities was based on automatic identification. Due to the precise drift correction, the actual error is limited by the pixel size, which is taken into our error estimation. We elaborate on this in the revised Supplementary Information:

The long acquisition time leads to electron-beam drift, primarily as small translational shifts between neighboring frames. To correct this, we precisely select ~10 robust image features per frame and perform a semi-automated feature-based registration of each reconstruction to compute the correction (Fig. S2). This number of features provides sufficient constraints for x–y drift and makes the transformation robust to localization errors. After alignment of all frames, we perform a linear interpolation to generate a movie with an effective temporal resolution of 0.2 fs (Movie S1). For a pixel size of 20 nm and a 3 fs sampling interval, a half-pixel misregistration corresponds to a velocity uncertainty of $\Delta v \approx 3.3 \cdot 10^{-6}\ \mathrm{m/s}$, which is approximately only 1% of the speed of light.

Minor Comments

- Abstract: Maybe replace “unbounded velocities exceeding the speed of light” with “divergent apparent velocities of phase singularities.”

We have changed this sentence to “toward formally divergent velocities in the moment before annihilation”.

- Fig. 2 caption: substitute “superluminal” with “divergent apparent” motion.

We have changed accordingly to “…acceleration toward formally divergent velocities along a characteristic space-time trajectory” and add “appears as” before superluminal later in the caption.

- Discussion (lines 312–323): superoscillations are conceptually distinct from superluminality; this section should not conflate them.

We agree with the referee that superoscillations and superluminality should not be conflated. The fact that “super” appears in both terms is a complete coincidence. We made sure to clarify this, and highlight that our experiment observes the two distinct phenomena separately (one in every frame and the other in the evolution between frames). We find it valuable to point to the prior literature discussing established connections between superoscillations and phase singularities (e.g., Ref. [43] argues that superoscillations inherently arise in the vicinity of phase singularities).

In accordance with the referee's suggestion, we have revised that paragraph about superoscillations to distinguish them from superluminality and explain the different way each is being observed:

The past decade has shown renewed interest in the superoscillatory[42-44,69,70] nature of certain wavefields—where local field/phase gradients can exceed the maximum spatial frequency. Superoscillations arise near the center of every phase singularity[71] and hold promise for advanced optical microscopy techniques with deep subwavelength resolution[41]. Our experiment observes simultaneously, and distinguishes, the two phenomena: (1) superoscillations are inherently created by phase singularities as seen at every frame; (2) the apparent superluminal velocities are observed in the dynamics between frames. There seems to be an indirect, yet intriguing, connection between these distinct features of topological defects, calling for further research: The extreme dynamics of singularities appear to be a direct manifestation of superoscillatory field gradients, once undergoing temporal evolution.

- In the methods: quantify calibration of sub-cycle timing (3 fs resolution) and conversion from phase delay to absolute time.

Timing is due to our delay stage (Newport XMS50-S), which controls the phase delay between the two interaction points. Another delay stage (Newport IMS-400CC) controls the overall time delay between the arrival of the electron and the excitation of the sample. The time resolution of 3 fs arises from our choice of delay stage step size of 0.9 µm. An uncertainty of 5% is propagated into the errors. We now clarify this in the Supplementary Information:

In our experiment, we set the time delay between frames to be 3 fs. This delay is controlled by the delay stage step size of 0.9 µm. An uncertainty of 5% is propagated into the errors.

- Fig. 4: indicate the number of events per bin and statistical weight of each region marked N/A.

We have updated accordingly the caption of Fig. 4:

The bins of the experimental results hold a min of 50 singularity counts per bin. N/A denotes regions where not enough singularities were found to generate trustworthy statistics, or where the velocities were too high for detection.

- Equation (4): unify notation for λ, k, and Δk.

We simplify the presentation in the main text under Eq. (4), expressing the final formulas using only wavelength λ_0 and its standard deviation $\Delta\lambda$, without introducing k and Δk at all.

Understanding the Result through Simple Models

To better understand the nature of the reported “superluminal” effects, I have examined a couple of simple theoretical models that reproduce similar behavior under fully causal assumptions. These include:

1. Interference between two slightly detuned waves, where the nodal lines sweep through space faster than c.

2. A phased antenna array whose programmed time delays generate a wavefront that appears to move superluminally across a distant screen.

In both cases, the field fronts themselves remain causal; only the geometric loci of constructive or destructive interference move faster than c. These toy models demonstrate why the measured singularity velocities are best viewed as apparent kinematic features of a random optical field, not as superluminal correlations in any physical sense.

We thank the referee for the detailed analysis. We added a section in the Supplementary Information (see Section XI, copied below), explaining the apparent superluminal velocity with subluminal information flow. We do this following the approach suggested by the referee, presenting a simple model creating a single singularity moving with velocity higher than c. A simple visualization of the model is provided in a new figure (see Fig.S5, copied below).

XI) Apparent superluminal motion with subluminal information flow

This section provides a toy model of singularity motion in a 2D wavefield, in order to help distinguish between the dynamics of the singularity from the kinematic pattern speed, phase velocity, motion of fringes, and dynamics of caustic ridges. All these differ further from the velocity of information flow and energy transport in the system. By using the term "apparent superluminal motion" throughout this paper, we mean that a geometric locus (e.g., a zero of ψ, a bright/dark fringe, or a caustic edge) can traverse space faster than c without conveying energy or information[18,19]. This separation is consistent with standard causality arguments: even where phase speeds exceed c, the information-bearing front remains subluminal. This is of course the case in our experiment.

Here we consider the simplest example when the singularity appears to be "superluminal". This section demonstrates that apparent "superluminal" motion of singularities arises naturally in the superposition of plane waves, even in simple configurations (Fig S5 (a)). We consider the superposition of three waves, with one detuned in frequency:

$$E(x,y,t)=e^{-i\omega t}\left(e^{ikx}+e^{ik(x\cos\theta+y\sin\theta)+\frac{2\pi}{3}i}+e^{i(k+\delta k)(x\cos\theta-y\sin\theta)+\frac{4\pi}{3}i}e^{-i\Delta t}\right),\qquad \text{(S31)}$$

where $k=n(\omega)\omega/\omega$, $\delta k\approx n(\omega)\Delta/c$. According to Eq. (S31), at time $t=0$ the singularity is located at the origin $(x=0;y=0)$. The theory in Ref.[16] enables to extract the velocity of the singularity:

$$\boldsymbol{v}=\frac{\left(\partial_t(\mathrm{Re}[E])\boldsymbol{\nabla}(\mathrm{Im}[E])-\partial_t(\mathrm{Im}[E])\boldsymbol{\nabla}(\mathrm{Re}[E])\right)\times\left(\boldsymbol{\nabla}(\mathrm{Re}[E])\times\boldsymbol{\nabla}(\mathrm{Im}[E])\right)}{|\boldsymbol{\nabla}(\mathrm{Re}[E])\times\boldsymbol{\nabla}(\mathrm{Im}[E])|^2}\qquad \text{(S32)}$$

Substituting into Eq. (S32) the field in Eq. (S31), we get:

$$|\boldsymbol{v}|=\frac{c}{n}\frac{(\Delta/\omega)}{\cos\theta/2\,|2(1-\cos\theta)+(\Delta/\omega)(1-2\cos\theta)|}\qquad \text{(S33)}$$

Considering a small detuning $\Delta/\omega\approx 0.02$, small angle $\theta=0.1$, and no medium $n=1$, we get:

$$|\boldsymbol{v}|=2.04c,\qquad \text{(S34)}$$

i.e. velocity of singularity is twice as high as the speed of light. Thus, we got singularity moving with apparent “superluminal” speed despite subluminal information flow.

Other representative examples of the same kinematic mechanism include: (i) Fast motion of nodes and other interference dislocations generated by weak detuning/tilt, or by spatiotemporal structuring. e.g., spatiotemporal optical vortices and ultrafast Rabi-rotating vortices whose singular cores move kinematically rather than by energy transport[23,24]. (ii) Superluminally swept illumination producing image-pair creation and annihilation as a purely geometric/light-in-flight effect with no superluminal signalling[25]. (iii) X-shaped localized waves whose peak (group) velocity can be engineered to be superluminal while the information-carrying front remains causal**Error! Reference source not found.**[20,21,22,26].

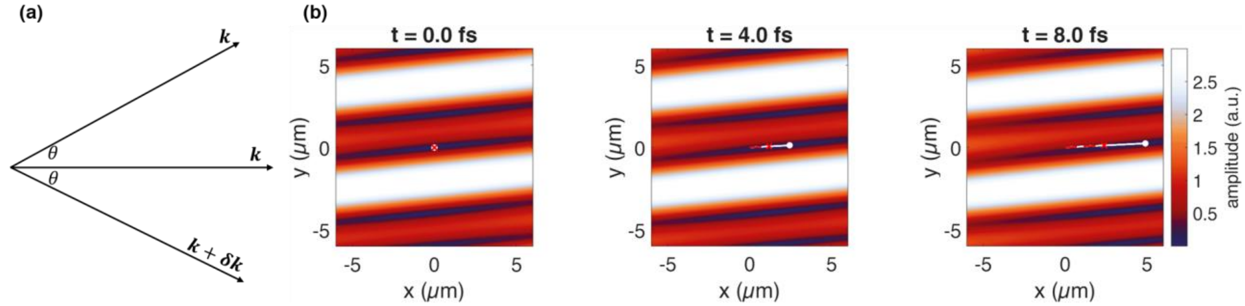

Fig. S5 | Superposition of three waves for a simple example of apparent superluminal singularity motion. **(a)** Superposition of three plane waves. **(b)** The corresponding field $E(x, y, t)$ at $t = 0{,}4{,}8$ fs. The white circle marks the singularity throughout time, moving at $|v| = 2.04c$. The red marker would be the singularity position if it were moving at $|v| = c$ along the same trajectory, highlighting that the singularity indeed moves superluminally.

Referee #2:

This work provides an experimental measurement of phase singularities in random scalar waves, verifying statistical properties like pair correlation, velocity probability densities, and distance-velocity correlation, explaining how nearby singularities of opposite charge can move faster than light. I have never seen real optical phase singularities drifting around over time and crashing into each other, so the supplementary video alone was very enjoyable to watch. Although I do not have strong experimental expertise, the authors provide an extensive account of their methodology across the main text and supplementary information and I found their presentation of results convincing.

The main strength of the work is the observation of singularities moving faster than light when created and annihilated in pairs and measurement of velocity and distance velocity probability distributions. The superluminal speed of phase singularities has been understood beyond much doubt in theory for a fairly long time (e.g., Ref. 8) so the presented results aren't surprising per se - but this does not diminish importance. Experimental verification also carries significant value as a technical achievement, in terms of the high simultaneous spatial and temporal resolution and reconstruction of the time-varying field. It ties in well with the current popularity of time-varying metaoptics and it is certainly of interest to researchers in optics. I believe it could have a significant impact simply as a demonstration of a way to observe the time-dynamics of delicate sub-wavelength features of light, as the authors conclude.

As the authors mention, phase singularities are a universal feature of structured waves and have a broad appeal beyond optics. I can also see the work's headline result (and supplementary video) being of interest to the general public. On this basis I would recommend publishing in Nature.

We thank the referee for the positive feedback and the recommendation for publication in Nature.

However, I would like the authors to address my queries below:

1. On line 50: The authors point out phase singularities "serve as carries of classical and quantum information". But later on line 158: "Phase singularities carry zero intensity and thus can move superluminally". The authors should reconcile these two statements a bit more carefully, e.g., explaining where the information is carried (i.e. not in the singularity core). Given this the authors should justify the statement of "suggesting phenomena of ultrafast information flow" in the abstract.

We thank the referee for this useful suggestion and we now fix the confusing statements in the revised manuscript. Phase singularities themselves carry no energy nor information, which is why there is no contradiction in having them move superluminally. We now clarify this at the beginning of the section under Fig. 1.

The acceleration of opposite-charged singularities before annihilation or after creation is a universal feature in the interference of Gaussian random waves and can be understood through the space-time trajectories of annihilating singularities (Fig. 2(a)). As opposite-charged singularities approach each other, their paths in space-time must form a continuous curve at the annihilation point, forcing their acceleration to unbounded velocities right before the annihilation. This fact is a mathematical consequence of the continuity of the phase rather than a violation of physical laws: Phase singularities do not carry energy or information and thus can "move" superluminally without breaking causality[13]; their apparent superluminal motion is a pure kinematic property of the evolving phase landscape. To better understand how singularities can move with unbounded velocities, in SI Section XI we show a simple example of such a phenomenon.

We also revise the statement about the ultrafast information:

Our findings deepen our understanding of phase-singularities and their universality, enabling to probe topological defect dynamics at previously unattainable timescales.

Despite these statements, optical singularities are used to encode information in a few optical platforms. The information is then encoded in the field surrounding the singularity, for example in its orbital angular momentum. This encoding does not contradict singularity motion with superluminal velocities, as the direction of motion should be considered as well (e.g., our singularities are moving transversely in-plane while common use cases in optical communications often involve out-of-plane information transfer). We add a clarification in the introduction line 48:

Optical singularities enable precise control of light-matter interactions with both bound[35-37] and free[38-40] electrons, underpin super-resolution imaging[41-44], and encode classical[45] and quantum information[46] that can be imprinted on the orbital angular momentum (OAM) carried by the field surrounding the singularity.

2. In the section starting on line 87: I understand the suitability of the experimental setup appropriate for making singularities observable, but why is it appropriate for generating random scalar waves in the first place? What produces the Gaussian randomness in the distribution of the phase singularities? Does it come from the hBN structure? Likewise, how do the time dynamics of the singularities relate to the excitation? This would provide useful context for someone minded more towards pure singular optics theory.

We thank the referee for these questions. We now update the manuscript to address these questions. In short, Gaussian random waves are created when multiple plane waves interfere, each with a random uniformly distributed phase [Berry, *Proceedings of the Royal Society of London A*, **456**, 2059-2079 (2001)]. In our case, when the laser pulse arrives at the hBN flake, it couples via the edges, launching phonon polaritons at different angles and phases from the rough boundaries. These polariton waves further undergo multiple scattering from the boundaries inside the flake. Together, these conditions create a complex interference pattern. Therefore, the creation of Gaussian random waves in our experiment is due to the boundaries of the hBN sample and the light coupling. We elaborate on this in chapter VI of SI.

Thus, the Gaussian randomness (i.e., interference of multiple plane waves with random phase) is created by rough boundaries of the sample. The statistical properties of the singularities such as velocity distribution are universal and independent on the material of the sample, hBN structure only enhances the average velocity of the singularities, allowing $\langle v \rangle \approx c$. The average velocity also depends on the wavelength of the excitation λ_0 and its standard deviation $\Delta\lambda$.

In the main text:

In our experiment, the rough boundaries of the hBN sample couple the incident laser into multiple PhP plane waves with arbitrary angles and phases. These waves then undergo multiple internal scattering at other boundaries, generating a complex interference patterns, producing Gaussian random wave statistics. Despite its simplicity, the random-waves model captures a wide range of universal wave phenomena and applies to diverse experimental settings. Within this framework, the dynamics of the singularities is universal across different wave systems; only the parameters of their statistical distributions (such as the average velocity) depend on specific material properties like wave dispersion and excitation properties like the central wavelength λ_0 and standard deviation (bandwidth) $\Delta\lambda$.

In the Supplementary Information, Section VI:

We describe the measurement using the theory of Gaussian random waves. Such random scalar waves arise from the interference of many plane waves with random, uncorrelated phases, as established in the theory of isotropic random waves[10,11]. In our setup, the hBN sample has rough boundaries causing multiple scattering and complex reflections. When excited by the incoming laser pulse, light couples at the edges to launch phonon polaritons that propagate at arbitrary angles and phases. The electrons measure the electric field component along their trajectory, extracting from the of the polariton interference pattern a scalar field in the plane of the hBN flake, obeying the statistics of Gaussian random waves.

Within this Gaussian random-wave framework, the qualitative dynamics of phase singularities is universal and does not depend on the specific material platform: different systems all exhibit the same characteristic behaviors, such as heavy-tailed velocity distributions, annihilation and creation events, and the same forms of spatial and temporal correlation functions. What does depend on the material are the numerical values of the parameters that set the scales of these distributions. In particular, quantities such as the average singularity velocity, the typical displacement over a given time interval, and the characteristic lifetime of singularity pairs are controlled by the excitaiton's central wavelength λ_0 and bandwidth $\Delta\lambda$, together with the group and phase velocities of the supported modes. These parameters determine the excited wave-vector spread and the relevant temporal scales, and thus simply rescale the universal distributions rather than changing their functional form.

3. On line 200: The authors say the correlation function g(R) is "defined as the probability of finding a pair of singularities at a distance R from one another" This seems imprecise because g(R) exceeds 1 - is it not a measure of the local density of singularities, rather than a probability? This also occurs on line 397 and in section VI of the supplementary information.

We thank the referee for spotting this mistake. Indeed, on line 200, $g(R)$ is not the raw probability, but the standard pair-correlation function: i.e. the probability density of finding a pair of singularities at separation R, normalised by the mean singularity density (and computed separately for same-charge and opposite-charge pairs). Thus $g(R) > 1$ correctly indicates local density higher than the global average, whereas $g(R) \approx 1$ corresponds to uncorrelated positions. We have revised the text at lines 200, 397, and in the Supplementary Information section VI. For example, in line 200 we now write:

Fig. 3(e) presents the experimentally measured and theoretically predicted distance correlation functions for the same charge $g_{+|+}(R) = g_{-|-}(R)$ and opposite charge $g_{+|-}(R) = g_{-|+}(R)$. Each distance correlation function is defined as the probability density of finding a pair of singularities at separation distance R, normalized such that $g(R) = 1$ corresponds to the global average density, for the same or opposite charge[7,8,61].

4. In the text around Figure 3 (and the figure's caption), descriptions for Fig. 3(d) and (e) seem to be the wrong way round.

We thank the referee for noticing this typo; we have corrected accordingly.

5. In Figure 4 - showing numerical colourbar limits would be useful.

We agree with the referee and add the colorbar limits in Fig. 4.

6. In the conclusion the authors suggest different platforms might provide a way to measure different kinds of singularity. Is there any realistic prospect for measuring the time-dynamics of vector (polarisation) singularities or larger scale topologies (like near-field optical skyrmions). Or is this what is meant by "multi-dimensional" singularities?

We thank the referee for suggesting this direction; we now expend on the idea of using our experimental platform for observing skyrmions in the outlook section:

Future work that will probe higher-order and multi-dimensional singularities provide a larger and richer space for encoding information[74,75], corresponding to more intricate types of collective phenomena in the overall system. In that sense, our approach could be applied to study other topological phenomena such as optical skyrmions[48], tracking their ultrafast dynamics and resolving their sub-cycle features. By using electron energy post-selection, one can further achieve orbital angular momentum selectivity[39]. Then, using an additional point of interaction for mixing different states of angular momentum will enable performing quantum tomography[76] of exotic topological states such as quantum skyrmions[77].

Referee #3:

The paper is devoted to tracking the dynamics of phase singularities near annihilation events, with the detection of short-term superluminal motion. An assessment of both individual annihilation events and a statistical analysis in phase space is proposed. And it is shown that the measured distance correlation agrees with the particle-like nature of singularities, and the distribution of velocities violates the corpuscular analogy. An experimental solution for assessing the dynamics of singularities and their statistical properties is proposed. The results are original and interesting.

We thank the referee for the overall positive review, helpful comments, and for stating that our work is interesting and original.

The article lacks attention to detail, which affects the overall understanding of the problem and leads to some uncertainty in the processing of the results. The conclusions are also not reliable enough. There are sufficient references to previous works.

But there are comments and remarks

We added clarifications and addressed point-by-point all the comments below.

-A physical approach to studying phase singularities is the interference of Gaussian random two-dimensional beams. What happens in the three-dimensional case if we are talking about the near field?

We thank the referee for raising this question. In three dimensions, interference of random waves results in *line* singularities, continuous *curves* along which the phase is undefined (rather than points). These singularity lines can also exhibit ultrafast motion and statistical correlations. Some of their properties generalize the correlations of point singularities while others are still under investigation. We now add a paragraph about these topological features and cite a review paper.

Experimentally imaging such line singularities is challenging, and recording their dynamics would be even harder, due to the limited ability of current ultrafast transmission electron microscopes to measure only the field projection along one axis. To access the full three-dimensional near-field information and its dynamics will require inventing new methods for tomographic reconstruction in ultrafast electron microscopy. A recent work by our group [Shpiro, arXiv:2510.24648 (2025)] proposed such a technique and developed the underlying theory, showing that such 3D measurements are in principle achievable.

We added a new paragraph to the outlook section and thank the referee again for proposing this.

This experiment studied the behavior of singularity in 2D random Gaussian waves. In 3D, interference of random waves results in line singularities (e.g., C lines[13]), continuous curves along which the phase is undefined. These singularity lines exhibit ultrafast motion and statistical correlations analogous to point singularities. Experimentally imaging such line singularities in the near field is extremely challenging due to the limitation of current ultrafast transmission electron microscopes to measure only the field projection along one axis. Probing the richer space of 3D singularities, and more generally 3D near-fields, requires developing tomographic reconstruction techniques in ultrafast electron microscopy[78].

-The conclusion is somewhat unusual and seems naive that the observation of the maximum velocity is “limited not by the field properties, but by the state-of-the art of the microscopy”. Although at the moment the advanced microscopy, subwavelength focusing, smaller than the diffraction limit, is manifested even in the formation of photonic jets and photonic zigzags.

We thank the referee for raising this question, which gives us an opportunity to further clarify this important point in the revised manuscript. The observation of the maximum velocity is indeed limited by the microscopy and not by the material. The reason is that the velocity of singularities in our system (and generally, in any Gaussian random wave) are formally, mathematically, *divergent*. The maximum observable velocity is only limited by the observation resolution. This seems paradoxical, but is a universal property of singularities, which does not break causality because singularities do not carry energy or information.

We revise the manuscript in multiple places to explain this. In the introduction:

As opposite-charged singularities approach each other, their paths in space-time must form a continuous curve at the annihilation point, forcing their acceleration to unbounded velocities right before the annihilation. This fact is a mathematical consequence of the continuity of the phase rather than a violation of physical laws: Phase singularities do not carry energy or information and thus can “move” superluminally without breaking causality[13]; their apparent superluminal motion is a pure kinematic property of the evolving phase landscape. To better understand how singularities can move with unbounded velocities, in SI Section XI we show a simple example of such a phenomenon.

Near Fig. 3:

Fig. 3(d) presents the ubiquitous particle analogy for singularities: As also shown before[7], the distance correlation function resembles those of particles that make up a liquid (with average period of correlations ~0.3 nm)[9], exhibiting similar spatial short-range order due to their interactions. Going beyond the distance correlations, we now also measure the velocity distribution (Fig. 3(f)), as proposed in Ref.[8]. The velocity distribution has a distinct heavy tail. Specifically, comparing to the (Maxwell–Jüttner) velocity distribution of particles at the same relativistic temperatures (Fig. 3(d) bottom) shows the unique behavior of singularities ensembles; their heavier tails, as discussed in Ref. [8], breaks the particle-singularity analogy.

After Eq. (4):

$\langle v\rangle$ is the average velocity of the singularities. This value is calculated by averaging all the measured velocities of singularities in the experiment (the distance that the singularity traveled between frames, divided by time step) and is measured directly in our experiment to be $\langle v\rangle \approx (1.04 \pm 0.04)c$.

The measured $\langle v\rangle$ is in close agreement with the theoretical prediction[8]: The average velocity is given by $\langle v\rangle = c \cdot \pi/\sqrt{2}\left(1 + (\lambda_0/\Delta\lambda)^2\left(v_{\mathrm{g}}/v_{\mathrm{ph}}\right)^2\right)^{-1/2}$, with v_{ph} and v_{g} are the average phase and group velocities of the PhPs (see SI, Section X). In hBN, the group velocity of PhPs is much smaller than the phase velocity over a broad-spectral range, with $v_{\mathrm{ph}}/v_{\mathrm{g}} \approx 12 \pm 1$ in our case, leading to a theoretical prediction of $\langle v\rangle \approx (1 \pm 0.1)c$, matching our measurement. This coincidence of having an average velocity so close to c further motivates the comparison of singularity velocities to the speed of light in Fig. 2.

In addition, we also revise and clarify the statement in the conclusions mentioned by the referee:

Finally, since the apparent velocities of phase singularities are formally divergent, their maximum observable value is limited by the state-of-the-art spatial and temporal microscopy resolution. Going beyond the current temporal resolution of few femtoseconds and spatial resolution of few tens of nanometers will enable observing singularity velocities orders of magnitude above c.

1. It is necessary to expand the paper with a deep physical interpretation of the obtained results with the velocity greater than the light velocity.

We completely agree with the referee about the need to provide a deeper physical interpretation of velocities exceeding the speed of light, as observed in our system. To this end, we have added a dedicated section in the Supplementary Information (section XI) illustrating a simple model and example: showing how this effect can emerge theoretically from the interference of just three waves in vacuum. We copy this section below.

We also expanded the explanations throughout the paper:

… Most notably, theory has long predicted that optical singularities exhibit heavy-tailed statistics, containing extreme events of apparent superluminal motion (singularities exceed the speed of light without breaking causality). …

… This fact is a mathematical consequence of the continuity of the phase rather than a violation of physical laws: Phase singularities do not carry energy or information and thus can "move" superluminally without breaking causality[13]; their apparent superluminal motion is a pure kinematic property of the evolving phase landscape. …

2. Phase retrieval has been implemented with the generalization of FERI theory. The explanation provided in the text is not sufficient to represent the generalization mechanism.

Following the referee's comment, we now expand the explanation of the FERI algorithm and its generalization in the revised manuscript. Variants of the FERI algorithm are used in recent works in the literature [Bucher, *Sci. Adv.* **9**, eadi5729 (2023); Baum, *Nature* **619**, 63-67 (2023); Ropers, *Nat. Phot.* **18**, 509-515 (2024); Bucher, *Nat. Phot.* **18**, 809-815 (2024)]. The first of these cited papers provided the full theory and analysis. We now direct from the main text to Section IV of the Supplementary Information which explains the algorithm.

In the main text we write:

The reference and sample pulses can be tuned independently, allowing control over their relative sub-cycle (phase) delay $\Delta\varphi$, intensity, and polarization. The amplitude and phase at each frame is acquired by measuring multiple sub-cycle delays $\Delta\varphi$ following the approach of free-electron Ramsey imaging (FERI) demonstrated in Ref.[30] and the algorithm presented in Ref.[29]. This way, we extract the phase and amplitude of the PhP near-field component pointing along the electron trajectory (Fig. 1(b,c)). See SI, Section IV for extended details.

In Supplementary Information Section IV, we now provide a complete stand-along description of the FERI theory:

IV) Free-electron Ramsey imaging (FERI): theoretical framework

At the core of PINEM lies the fundamental interaction between free electrons and near fields. Following typical approximations such as electron beam paraxiality[4,5,6,7,8] , the Hamiltonian describing the interaction between the electromagnetic field and the electron is given by $H = E_0 - i\hbar v\partial_z + evE_z(x,y,z)/\omega$, where E_0 is the initial electron energy, v is the electron velocity, e is the fundamental charge, z is the electron propagation direction, $E_z(x,y,z)$ is the local electric near-field amplitude along z, and ω is the electromagnetic field fundamental frequency. The interaction is then governed by the dimensionless parameter $g(x,y) = \frac{e}{\hbar\omega}\int_{-\infty}^{\infty} dz E_z(x,y,z)e^{-\frac{iz\omega}{v}}$. In general, $g = g(x,y)$ is a complex number and a function of the transverse coordinates xy. By measuring $g(x,y)$, one can, in principle, recover both the amplitude and the phase of the near field along the electron trajectory $E_z(x,y)$.

The probability for the electron to change its energy by $l\hbar\omega$, where l is an integer, is given by $P_l = |J_l(2|g|)|^2$, where J_l is the l-th order Bessel function of the first kind. Furthermore, considering the electron energy spread ΔE and its Gaussian profile $G_{\Delta E}(E) \propto e^{-\frac{E^2}{\Delta E^2}}$, we express the combined (measured) electron energy distribution as:

$$P(E) = \sum_l |J_l(2|g|)|^2 \, G_{\Delta E}(E - l\hbar\omega)\,. \tag{S1}$$

By considering two points of interaction, we have $g = g_\mathrm{r} + g_\mathrm{s}$, where and g_r and g_s are the interaction strengths of the reference and sample fields, respectively. The FERI theory can be directly generalized for cases where the distance between the two points of interaction is large (and hence the electron pulse dispersion must be accounted for)[9]. However, this is not necessary in our setup since the distance between the two interaction points is 37 mm, negligible compared to the Talbot distance, which is in the order of a few meters in the case of mid-IR. This allows us to neglect the electron pulse dispersion between the two points of interaction. The electron then performs an effective interference between the reference and sample near-fields[10]. To measure $P(E)$, the electron pulse is sent through a dispersive magnetic prism and filtered in energy using a slit to extract only the part of the electron that after both interactions have energy within $E_\mathrm{slit} \in [-E_\mathrm{min}, E_\mathrm{max}]$.

Using Eq. (S1), the theoretical model of the signal in energy-filtered TEM (EFTEM) is:

$$M\big(x,y,t,\Delta\phi,E_\mathrm{slit},g_s(x,y)\big) =$$

$$= \int_{E_\mathrm{min}}^{E_\mathrm{max}} \sum_l G_{\Delta E}(E - \hbar\omega l) \cdot |J_l(2|g_\mathrm{r}(\Delta\phi) + g_\mathrm{s}(x,y,t)|)|^2 \, dE \tag{S2}$$

where x, y are the sample spatial coordinates, t follows the evolution of the sample dynamics, $\Delta\phi$ is the relative phase between the two laser pulses inducing the two interactions, and $E_\mathrm{slit} = [E_\mathrm{min}, E_\mathrm{max}]$ is the electron energy-filtering range. Repeating this measurement while varying the sample-reference relative phase $\Delta\phi$, using sub-cycle steps, allows us to generate a complete dataset (phase scan), which fully captures the (z component) near-field dynamics at every transverse point of the sample.

The FERI algorithm**Error! Reference source not found.** then reconstructs the phase and amplitude of the field at every point of the sample from the energy filtered measurements of the phase scan. The reconstruction minimizes the following expression:

$$\underset{|g_s(x,y)|,\angle g_s(x,y)}{\operatorname{argmin}} \sum\nolimits_{\Delta\phi} \left| Y_i(x,y,\Delta\phi,E_{\mathrm{slit}}) - M\big(x,y,t,\Delta\phi,E_{\mathrm{slit}},g_s(x,y)\big) \right|^2 \qquad \text{(S3)}$$

where $Y_i(x,y,\Delta\phi,E_{\mathrm{slit}})$ is the i-th experimental measurement of the phase scan $\Delta\phi$ for energy filtered in E_{slit}, M is the measurement model (see Eq. (S2)). Measuring the spectrum for each $\Delta\phi$, and using maximum likelihood estimation, we minimize the Eq. (S3), finding the amplitude $|g_s(x,y)|$ and phase $\angle g_s(x,y)$ of the sample field for each point in space independently.

This optimization expression is not convex. That is, it exhibits multiple local minima, which limits the applicability of the standard gradient descent methods that often converge to a local minimum rather than the global minimum. To avoid this issue, the minimization procedure scans over the entire range of possible relative phases $\Delta\phi$, and owing to the low variable space (solving for just two unknowns, $|g_s|$ and $\angle g_s$), we are able create a heat map and perform an exhaustive search, for which finding the global minimum is guaranteed.

3. How is longitudinal (transversal) coherence taken into account under all these FERI transformations?

The big advantage of FERI is that the transverse coherence is not needed at all. Unlike other phase-sensitive methods in electron microscopy, FERI relies solely on longitudinal electron modulation. In fact, the longitudinal coherence is itself only needed in the "quantum" regime of FERI [Bucher *Sci. Adv.* **9**, eadi5729 (2023)]. In this experiment, using mid-IR frequency puts us in the "classical" regime of FERI, where longitudinal coherence is not required. Instead, what is required is having the same electron synchronized in its arrival time to the laser excitations at the two interaction points. The electron longitudinal modulation (time-dependent changes in its energy) is what mediates the interferometric measurement of the polaritonic phase.

This is precisely what differentiates FERI from conventional phase-retrieval or interferometric electron-optics methods (e.g., holography), where transverse coherence is the essential resource. In these conventional methods, limited beam quality suppresses high spatial frequencies. In FERI, transverse coherence is not a prerequisite. The algorithm does not suffer the usual resolution loss linked to limited transverse coherence.

We now elaborate on this point in the main text:

We used free-electron Ramsey imaging (FERI)[29] to reconstruct the complex near field at the sample. By scanning the relative optical phase between reference and sample interactions in sub-cycle steps, and by recording energy-filtered images, we retrieved amplitude and phase per pixel via an optimization-based forward model of the PINEM/FERI interaction (see SI IV for extended details). In contrast to most phase-contrast imaging methods and phase retrieval methods in electron microscopy that rely on transverse coherence, FERI does not require transverse coherence at all. Instead, it relies on the longitudinal (temporal) energy modulation of the electron encoded through the PINEM interaction. This longitudinal modulation is a robust resource that is routinely achieved in ultrafast electron microscopes, as demonstrated by the widespread observation of PINEM sidebands[**Error! Reference source not found.**,23,24,25,26,27,28,30,31].

We also update the Supplementary Information (added as last paragraph to the section copied above):

Our high spatial resolution is achieved here without using any spatial coherence of the electron beam despite. This aspect is worth emphasizing as it relates to the operation of the FERI method. Most phase-retrieval and phase-imaging approaches in electron microscopy rely on transverse coherence; consequently, they lose high-spatial-frequency features when the transverse coherence of the electron is limited (e.g., due to finite emittance, source quality, etc.). In contrast, FERI does not rely on transverse coherence at all. Instead, it leverages the longitudinal (temporal) modulation of the electron through the PINEM interaction, which produces phase-sensitive interference between energy–time components. This longitudinal modulation is more robust because it is governed primarily by the laser's temporal coherence rather than by the coherence of the electron beam. The widespread observation of PINEM sidebands in ultrafast electron microscopes directly confirms that the required electron properties are available in practice[4,5,6,7,8]. We showed here and in previous works [21,22,23] that even without observing the sidebands, PINEM-based measurements (and thus also FERI) remain effective. This point implies that our technique is applicable for simpler, higher-flux electron sources[24**Error! Reference source not found.**], also in scanning electron microscope energies[25]. This robustness is what allows FERI to preserve high-spatial-frequency information and maintain high resolution for time-resolved singularity tracking.

It is absolutely unclear the influence of polarization of incident beam. How is controlled the intrinsic properties of incident beam, temperature changes in the sample under beam action and heating?

We now discuss the polarization of incident beam, as well as the sample heating in the Supplementary Information, Section 1:

To control the polarization of the incident IR pulse, we use a polarizer, creating a TM polarization. Then the pulse is focused using two lenses positioned near the microscope column, reaching a spot size of ~100 μm and an average power of 4-12 mW at the hBN sample. At the reference sample (electron pre-modulation stage), the pulse should be uniform across its interaction with the electron beam, and thus its spot size is larger, ~500 μm with an average power of 4-20 mW.

Given the high in-plane thermal conductivity of hBN[1], the laser-induced temperature rise of the sample is estimated to be only a few Kelvin. This heating effect is negligible in its influence on the PhP dispersion, and it also remains far below the damage threshold of hBN, ensuring that the measured dynamics is not significantly influenced by thermal effects.

The term "spatial correlation" was introduced, although its application to singularity correlation and annihilation was conceptually flawed.

The term "spatial correlation" for complex wavefields/random waves was originally defined by Berry and Dennis in [Berry, *Proceedings of the Royal Society of London A* **456**, 2059-2079 (2001)]. Following works used the same definition (e.g., [De Angelis, *Phys. Rev. Letters* **117**, 093901 (2016)]) and we keep the same terminology in our manuscript.

We now clarify the use of this term in the revised manuscript, writing:

Fig. 3(e) presents the experimentally measured and theoretically predicted distance correlation functions for the same charge $g_{+|+}(R) = g_{-|-}(R)$ and opposite charge $g_{+|-}(R) = g_{-|+}(R)$. Each distance correlation function is defined as the probability density of finding a pair of singularities at separation distance R, normalized such that $g(R) = 1$ corresponds to the global average density, for the same or opposite charge[7,8,61].

Let us also add that “spatial correlation” only represents the statical correlations and cannot describe the creation and annihilation of individual singularities. To study the creation and annihilation events, besides direct observation (e.g., Fig. 2), we also introduce the joint distance-velocity distribution. This distribution provides insight on events of creation and annihilation.
We clarify it in the main text:

The joint distance-velocity distribution, defined by Eq. (6), can capture the singularity-pair creation and annihilation events. Fig. 4 shows that at small distances ($R < \lambda_0$), the fraction of created and annihilated singularity pairs is large, leading to higher overall values of $P_{+|-}(v, R)$ and a wider variance, indicating higher possible velocities. The complementary $P_{+|+}(v, R)$ is smaller, since same-charge singularities are less likely at small distances, as expected by the known instability of singularities with charges higher than "±1"[52,53]. At larger distances, singularities cannot be created or annihilated, resulting in narrower velocity distributions (also see SI, Fig. S3).

The term “dynamical statistical properties of singularity pairs” is also introduced poorly.

We now avoid using this term in the revised manuscript. We use instead the terms: “spatial correlations”; “dynamics of singularities”; “distance-velocity distribution” or equivalently “phase-space correlations” that are all properly defined. We changed the main text accordingly.
Specifically, the text quoted by the referee is changed to:

This large dataset allows us to quantify the statistical properties and correlations among all singularity pairs: observing both their expected spatial correlations and previously inaccessible velocity correlations, which reveal universal properties in the collective dynamics of singularity ensembles.

The paper requires significant revision for possible publication

We addressed all the points raised by the referee, which helped improve the quality and readability of our work. We are grateful to the referee for these constructive comments.

Referee Report for Manuscript:

"Superluminal Correlations in Ensembles of Optical Phase Singularities"

General Assessment

The manuscript reports ultrafast imaging of optical phase singularities in hexagonal boron nitride (hBN) using a free-electron interferometric ultrafast transmission electron microscope (UTEM). The authors claim to directly observe *"superluminal" velocities* of phase singularities and to measure full phase-space correlations (distance–velocity joint distributions) in a singularity ensemble. The experimental resolution (20 nm spatial, 3 fs temporal) is impressive, and the technical implementation of phase-resolved UTEM represents a significant advance.

However, the central physical claim of *superluminal correlations* is conceptually misleading. The reported velocities correspond to the apparent motion of phase zeros, not to any transport of energy or information (as correctly pointed out in the manuscript). This makes the term "superluminal" in the title inappropriate and indeed misleading in many ways that could be detrimental to the otherwise serious approach taken in the manuscript. The results instead demonstrate ultrafast correlation mapping of topological defects, an important and rigorous result on its own. I find that this work stands as an elegant and rigorous confirmation of long-predicted heavy-tailed statistics in singular optics, rather than a claim of superluminal phenomena. Also, the authors correctly present their findings as an *experimental confirmation* of a long-standing prediction, not as the discovery of a new phenomenon.

Major Comments

The main claims of this paper revolve around "superluminal correlations" and the observation of singularities moving faster than light. While the experimental methodology is excellent, the interpretation of these findings is problematic, largely due to a misleading use of the term "superluminal" and an unclear notion of what the authors mean by "correlation." Below, I expand on these conceptual issues and discuss why the title and framing of the work should be revised. To clarify these points, I also include in the Appendix several analytical examples and simple models that reproduce such apparent effects within fully causal physics. I used them to clarify to myself how these superluminal zeros might arise from. These examples help make sense of the data and confirm that the reported heavy tails in the velocity distribution are both expected and interesting, but not acausal.

Misleading Claim of "Superluminal" Correlations

The manuscript repeatedly refers to "velocities exceeding the speed of light" (see lines 30–33, 157–159) and describes these as "superluminal correlations." However, the text itself acknowledges that phase singularities carry zero intensity and therefore can "move superluminally without energy or information transmission." This is indeed correct, the measured quantity is a *geometric velocity of the phase field*, not a physical or causal propagation speed. What is observed is the motion of field zeros in a complex interference pattern, a purely kinematic property of the evolving phase landscape.

It is therefore not clear what "correlations" are being described as superluminal. If this term is meant to denote a correlation function of singularity positions or velocities, then the adjective "superluminal" is misleading and unnecessary. No causation propagates faster than light, what is shown is that apparent velocities in the phase-space statistics can exceed c, which is a known property of random-wave fields.

For this reason, the current title exaggerates the phenomenon and risks confusion with genuine superluminal signal transport. A more appropriate framing would emphasize the remarkable experimental ability to resolve ultrafast correlations in ensembles of singularities, rather than to suggest a violation of causality. I would therefore recommend a title along the lines of:

> "Ultrafast Correlations in Ensembles of Optical Phase Singularities."

Also in the abstract and conclusion (lines 40, 319–323) further suggest that these measurements point to "ultrafast information flow." This interpretation is unsupported by the data. No energy, entropy, or causal signal has been shown to propagate faster than c. The observed dynamics instead represent the topological rearrangement of zeros in a continuous optical field, a process that can exhibit formally unbounded apparent velocities without transmitting any information. This speculative link to "information flow" should therefore be removed, or at least substantially toned down.

Also, I have a doubt that I wish the authors to clarify. The authors report an average singularity velocity $\langle v\rangle \approx 1.04\,c$ (lines 240–255), obtained by relating the phase and group velocities of polaritons in hBN ($v_\phi/v_g \approx 12$). This number is not a physically meaningful speed; it arises from parameters of a random-wave model in which the normalization by the small group velocity artificially amplifies apparent phase gradients. Am I wrong? If not, this should be clarified.

The statement that slow group velocity "amplifies" superluminal motion misrepresents the mathematics, the effect is statistical, not dynamical. It reflects the well-known result that the conditional distribution of the zero-velocity $\dot{\mathbf{r}}_0 = -\partial_t A/\nabla A$ has heavy tails that can extend beyond c without any energy transport.

The presence of these heavy tails in the experimental velocity distribution (Fig. 3f) is in fact quite interesting. While one might initially suspect they arise from tracking noise or phase-unwrapping artifacts, theoretical models of random-wave ensembles predict precisely such non-Gaussian tails. Their experimental confirmation is therefore of genuine significance, as it validates a statistical feature that had remained difficult to observe directly.

Experimental Uncertainties

The automated tracking of approximately fifty singularities across 285 frames is technically impressive. However, the drift-correction scheme (Extended Data Fig. 2) relies on manually selected features and may not guarantee sub-pixel accuracy. Residual misalignments could produce small artificial apparent velocities. Because of this, it is essential to propagate uncertainty through to the velocity distribution and to include confidence intervals for the "superluminal" fraction. Without such an analysis, the statistical significance of the long-velocity tail cannot be properly assessed. I am not sure whether this was taken into account in the error.

Minor Comments

- Abstract: Maybe replace "unbounded velocities exceeding the speed of light" with "divergent apparent velocities of phase singularities."
- Fig. 2 caption: substitute "superluminal" with "divergent apparent" motion.
- Discussion (lines 312–323): superoscillations are conceptually distinct from superluminality; this section should not conflate them.
- In the methods: quantify calibration of sub-cycle timing (3 fs resolution) and conversion from phase delay to absolute time.

- Fig. 4: indicate the number of events per bin and statistical weight of each region marked N/A.
- Equation (4): unify notation for λ, k, and Δk.

Understanding the Result through Simple Models

To better understand the nature of the reported "superluminal" effects, I have examined a couple of simple theoretical models that reproduce similar behavior under fully causal assumptions. These include (i) interference between two slightly detuned waves, where the nodal lines sweep through space faster than c, and (ii) a phased antenna array whose programmed time delays generate a wavefront that appears to move superluminally across a distant screen. In both cases, the field fronts themselves remain causal; only the geometric loci of constructive or destructive interference move faster than c. These toy models demonstrate why the measured singularity velocities are best viewed as *apparent kinematic features* of a random optical field, not as superluminal correlations in any physical sense.

"Superluminal" wavefront's zeros

This is one way in which I understand the result. Consider two waves of slightly different frequencies and wavevectors:

$$E(x,t) = \sin(k_1 x - \omega_1 t) + \sin(k_2 x - \omega_2 t), \tag{1}$$

with $k_2 = k_1 + \Delta k$ and $\omega_2 = \omega_1 + \Delta\omega$. Each wave individually satisfies a causal dispersion relation $\omega_i = v_{\phi,i} k_i$, but their slopes may differ because of material dispersion or phase-delay tuning.

Using the trigonometric identity for the sum of sines,

$$E(x,t) = 2\cos\left(\frac{\Delta k\, x - \Delta\omega\, t}{2}\right)\sin(k_{\text{avg}} x - \omega_{\text{avg}} t)\,, \tag{2}$$

where $k_{\text{avg}} = (k_1 + k_2)/2$ and $\omega_{\text{avg}} = (\omega_1 + \omega_2)/2$.

The amplitude envelope (cosine term) vanishes when

$$\frac{\Delta k\, x - \Delta\omega\, t}{2} = \frac{\pi}{2} + n\pi, \tag{3}$$

yielding the node trajectories

$$x_n(t) = \frac{\Delta\omega}{\Delta k}\, t + \frac{(2n+1)\pi}{\Delta k}, \qquad v_{\text{zero}} = \frac{dx_n}{dt} = \frac{\Delta\omega}{\Delta k}. \tag{4}$$

In vacuum, $\omega_i = c k_i$ and thus $\Delta\omega = c\,\Delta k$, so $v_{\text{zero}} = c$. If one component is detuned, for example, through dispersion in a medium with $v_{\phi,2} \neq v_{\phi,1}$—then $\Delta\omega/\Delta k > c$, producing an apparent *superluminal* motion of the interference zeros. This "detuning" means that the two waves no longer share an identical linear dispersion slope, not that either wave violates the light-speed limit. The nodes carry no field energy ($E = 0$) and thus transmit no information, preserving causality, as stated in the manuscript.

"Superluminal" Wavefront from a Timed Linear Array

Consider an idealized, thin, linear aperture on the x-axis ($x \in [-L/2, L/2]$) in free space. Each point x emits the same short pulse $s(t)$, but with a programmed emission time

$$t_{\text{emit}}(x) \;=\; \frac{x}{u}, \qquad u > 0, \tag{5}$$

so that a "trigger" *appears* to move along $+x$ at speed u. We allow $u > c$ (programmed timing), which does *not* imply any superluminal signaling between the elements.

Let $\mathbf{r}$ be an observer in the far field at polar angle θ relative to $+x$ (i.e. $\hat{\mathbf{r}} \cdot \hat{x} = \sin\theta$) and range $R \gg L$. To leading order in L/R, the retarded time from source element x is

$$t_{\text{ret}}(x,\theta) \;=\; t_{\text{emit}}(x) + \frac{R - x\sin\theta}{c} \;=\; \frac{x}{u} + \frac{R}{c} - \frac{x\sin\theta}{c}. \tag{6}$$

The observed field (scalar model) is proportional to the Huygens integral

$$E(\theta,t) \;\propto\; \int_{-L/2}^{L/2} A(x)\; s\big(t - t_{\text{ret}}(x,\theta)\big)\, dx, \tag{7}$$

with a slowly varying aperture taper $A(x)$.

For a short pulse s, the integrand is largest when the retarded-time argument vanishes:

$$t \;=\; \frac{R}{c} \;+\; x\left(\frac{1}{u} - \frac{\sin\theta}{c}\right). \tag{8}$$

Solving (8) for the contributing x at fixed (θ,t) gives

$$x(\theta,t) \;=\; \frac{t - R/c}{\frac{1}{u} - \frac{\sin\theta}{c}}. \tag{9}$$

Two key consequences follow:

(i) Caustic (Mach-cone–like) angle. When the denominator approaches zero,

$$\frac{1}{u} - \frac{\sin\theta}{c} \to 0 \quad\Longrightarrow\quad \sin\theta_\star = \frac{c}{u}. \tag{10}$$

For $u > c$ this has a real solution $\theta_\star \in (0,\pi/2)$. Near $\theta_\star$ the mapping $x \mapsto t$ becomes singular so that many nearby x contribute *nearly simultaneously*, producing a sharp, bright wavefront (a fold caustic) sweeping across angles θ. This is the precise sense in which the far-field "wavefront" can exhibit superluminal *apparent* motion.

(ii) Apparent angular/transverse speed. At fixed source point x, differentiate (8) (holding x constant):

$$1 \;=\; x\left(\frac{\cos\theta}{c}\right)\frac{d\theta}{dt} \quad\Longrightarrow\quad \frac{d\theta}{dt} \;=\; \frac{c}{x\,\cos\theta}. \tag{11}$$

On a sphere of radius R, the transverse speed of the luminous front is

$$v_\perp(\theta) \;=\; R\,\frac{d\theta}{dt} \;=\; \frac{c\,R}{x\,\cos\theta}. \tag{12}$$

As the dominant $x(\theta,t)$ passes near where the denominator of $x(\theta,t)$ is small (i.e. near $\theta_\star$), the apparent $v_\perp$ can greatly exceed c (and even diverge in the idealized, continuous-aperture limit). No information is conveyed by this apparent sweep; it is a locus of constructive interference formed from fields radiated earlier from *independent, locally driven* elements.

Causality. For any fixed observer (R,θ), the first nonzero field can arrive no earlier than R/c (from the nearest element edge), and *every* contribution obeys a retarded-time relation. Programming $t_{\text{emit}}(x)$ in advance cannot transmit a message faster than c, because after the array has begun radiating, any *new* modulation applied at element x_0 is seen at (R,θ) no earlier than $t \geq t_{\text{apply}} + |\mathbf{r} - \mathbf{r}_{x_0}|/c$.

B. Discrete N-element array with progressive time delay

Let N isotropic elements be placed at $x_n = nd$, $n = 0, \ldots, N-1$. Each radiates a baseband pulse $s(t)$ with a progressive delay

$$t_n \;=\; n\,\tau, \qquad \text{so that the apparent trigger speed along the array is } \; u \equiv \frac{d}{\tau}. \tag{13}$$

The (narrowband) passband field in the far field at angle θ is approximated by

$$E(\theta, t) \;\propto\; \sum_{n=0}^{N-1} s\Big(t - n\tau - \frac{R - nd\sin\theta}{c}\Big). \tag{14}$$

Pulses from element n arrive peaked when the argument of s in (14) vanishes:

$$t \;=\; \frac{R}{c} \;+\; n\Big(\tau - \frac{d\sin\theta}{c}\Big). \tag{15}$$

For increasing t, the index n that contributes to a given θ changes so as to keep the bracket small. Hence the *beam maximum* (or pulse front) *sweeps* across θ, and the sweep is fastest when $\tau \approx d\sin\theta/c$. Defining $u = d/\tau$, the condition $\tau - d\sin\theta/c \to 0$ becomes

$$\sin\theta_\star \;=\; \frac{c}{u}, \tag{16}$$

the same superluminal-caustic angle as in the continuous case. Near $\theta_\star$, the beam/spot can traverse angles on a distant screen with arbitrarily large *apparent* transverse speed $v_\perp = R\,d\theta/dt \gg c$, limited in practice by N, bandwidth, and tapering.

Each term in (14) is evaluated at a retarded time; no observer sees any influence before their light-cone time.

Superluminal sweep requires pre-scheduled $\{t_n\}$. A *new* bit imposed at n_0 after start reaches the observer no sooner than $|\mathbf{r} - \mathbf{r}_{n_0}|/c$. The sweeping wavefront (or bright line/spot) is a *locus* of constructive interference, not a material or energetic entity. Its kinematics are not bounded by c.