Low-Velocity-Favored Transition Radiation

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(Received 29 November 2022; accepted 10 August 2023; published 13 September 2023)

When a charged particle penetrates through an optical interface, photon emissions emerge—a phenomenon known as transition radiation. Being paramount to fundamental physics, transition radiation has enabled many applications from high-energy particle identification to novel light sources. A rule of thumb in transition radiation is that the radiation intensity generally decreases with the decrease of particle velocity $v$; as a result, low-energy particles are not favored in practice. Here, we find that there exist situations where transition radiation from particles with extremely low velocities (e.g., $v/c < 10^{-3}$) exhibits comparable intensity as that from high-energy particles (e.g., $v/c = 0.999$), where $c$ is the light speed in free space. The comparable radiation intensity implies an extremely high photon extraction efficiency from low-energy particles, up to 8 orders of magnitude larger than that from high-energy particles. This exotic phenomenon of low-velocity-favored transition radiation originates from the interference of the excited Ferrell-Bereman modes in an ultrathin epsilon-near-zero slab. Our findings may provide a promising route toward the design of integrated light sources based on low-energy electrons and specialized detectors for beyond-standard-model particles.

DOI: 10.1103/PhysRevLett.131.113002

Transition radiation occurs whenever a charged particle moves across an inhomogeneous region [1–8]; as shown in Fig. 1. One unique feature of transition radiation is that its radiation intensity is linearly proportional to the Lorentz factor $\gamma = (1 - v^2/c^2)^{-1/2}$ [9], if the particle velocity $v$ approaches the light speed $c$ in free space. This feature lays the foundation for many applications, including transition radiation detectors [10–12], useful for the identification of particles with extremely high momenta (e.g., $P > 100$ GeV/c or $\gamma > 10^5$) [9], as well as advanced light sources at the terahertz, ultraviolet, and x-ray regimes [13–21]. However, all these transition-radiation-based devices rely on high-energy particles, whose generation requires a giant and complex acceleration infrastructure and thus hinders the enticing on-chip applications of transition radiation.

Another feature of transition radiation is that its occurrence has no specific requirements on the particle velocity [1–8]. This feature is distinct from Cherenkov radiation [22–27], which occurs when a charged particle moves inside a homogeneous material with a velocity exceeding the phase velocity of light, namely the Cherenkov threshold [28–32]. As such, all applications of Cherenkov radiation are limited by the Cherenkov threshold, despite its enormous applications [33–39], including particle detectors, light sources, imaging, and photodynamic therapy. By contrast, transition radiation is applicable to develop novel light sources without fundamental restrictions on the particle velocities. However, the application of transition radiation based on low-energy particles remains largely unexplored. One reason is that the transition-radiation intensity generally decreases when the particle velocity...
low velocity (e.g., Berreman modes, low-energy particles with an extremely small velocity) remains unknown. Enhance the transition radiation from low-energy particles (e.g., free electrons) by exploiting the Ferrell-Berreman mode, which was first identified by Ferrell in 1958 in metal films at the ultraviolet regime [40] and later separately discussed by Berreman in 1963 in cubic ionic films at the midinfrared regime [41]. This mode is intrinsically radiative and appears near the frequency at which the relative permittivity of materials approaches zero. Moreover, the Ferrell-Berreman mode has enabled many applications [42–45] from imaging, sensing, to thin-film characterization. The transition radiation of Ferrell-Berreman modes has also been extensively studied since 1958 [4,46–49]. However, among these studies, the intensity dependence of the emitted Ferrell-Berreman mode on the particle velocity has been rarely discussed so far. Moreover, whether the Ferrell-Berreman mode can largely enhance the transition radiation from low-energy particles remains unknown.

Here, we find that due to the excitation of Ferrell-Berreman modes, low-energy particles with an extremely low velocity (e.g., $v/c < 10^{-3}$) could emit equally strong transition radiation as high-energy particles ($v/c = 0.999$). Consequently, the photon extraction efficiency from low-energy particles could be 8 orders of magnitude larger than that from high-energy particles, while the intensity of transition radiation from these particles is the same. This exotic phenomenon of free-electron radiation is then denoted as low-velocity-favored transition radiation, which is in a similar rationale but fundamentally different from low-velocity-favored Smith-Purcell radiation [50]. The revealed low-velocity-favored transition radiation indicates a promising route to enhance the particle-matter interaction, which may be exploited to design specialized detectors for beyond-standard-model particles with extremely low kinetic energy (e.g., detection of unknown millicharged dark matter [51–53]) and integrated light sources from low-energy electrons.

We begin with the introduction of transition radiation; see derivation in Supplemental Material, Sec. S1 [54]. An electron moves along the $+\hat{z}$ direction and perpendicularly penetrates through a thin epsilon-near-zero slab with a thickness $d$ [Fig. 1]. The epsilon-near-zero slab (namely, region 2), for example, is constructed by hexagonal boron nitride (BN) [55–61] with a relative permittivity of $[\epsilon_\perp, \epsilon_\parallel, \epsilon_z]$, which has $\epsilon_z \to 0$ around 24.5 THz. Both the superstrate (region 1) and the substrate (region 3) are free space with a relative permittivity of $\epsilon_1 = \epsilon_3 = 1$.

FIG. 1. Schematic of low-velocity-favored transition radiation from a uniaxial epsilon-near-zero (ENZ) material. A swift electron perpendicularly penetrates through a BN slab with a relative permittivity of $[\epsilon_\perp, \epsilon_\parallel, \epsilon_z]$, where $|\epsilon_z| \to 0$ near 24.5 THz. Unless specified otherwise, the material loss is considered in this work. Both the propagating waves and phonon polaritons would be excited during the electron’s penetration. Below, we focus on the light emission propagating into the far field, which is irrelevant to the excited phonon polaritons.

FIG. 2. Frequency spectral feature of low-velocity-favored transition radiation. (a) Radiation spectrum $W(\omega)$ of the excited propagating waves as a function of the electron velocity $v$ and the frequency. (b) Radiation spectrum as a function of $v$ at three representative frequencies. Here and below, when $\omega/2\pi = 24.5$ THz, the value of $W(\omega)$ at $v = v_A$ or $v = v_B$ is defined to be 90% of the maximum within the range of $v \in [v_A, v_B]$. (c),(d) Photon extraction efficiency $\eta(\omega) = W(\omega)/(\hbar \omega \cdot E_k)$, where $E_k$ is the kinetic energy.
Within the framework of macroscopic Maxwell equations, the induced radiation fields in regions 1 and 3 can be calculated, and the total angular spectral energy density of transition radiation is obtained as $U(\omega, \theta) = U_1(\omega, \theta) + U_3(\omega, \theta)$, where $\theta$ is the radiation angle between the wave vector of excited propagating waves and $\mathbf{v}$ (or $-\mathbf{v}$) for the backward (forward) radiation. $U_1(\omega, \theta)$ and $U_3(\omega, \theta)$ are the angular spectral energy densities of backward and forward radiation, respectively, whose calculation includes the light emission from the interface and that from the bulk. Accordingly, the total energy spectrum can be expressed as $W(\omega) = \int_0^{\pi/2} U(\omega, \theta)(2\pi \sin \theta) d\theta$. The detailed calculation of angular spectral energy densities and energy spectra of excited propagating waves is provided in Supplemental Material, Sec. S2, and their calculation is not related to the excited guided modes (e.g., BN’s phonon polaritons [55–58]) and is thus not necessary to apply the Sommerfeld integration [3,5,62–64].

As shown in Fig. 2(a), $W(\omega)$ is not only a function of the particle velocity but also sensitive to the frequency, due to the dispersive nature of BN. The BN thickness is 1 nm; see Supplemental Material, Sec. S1 [54]. Figure 2(a) shows that the frequency spectral feature of transition radiation near the frequency with $\varepsilon_z \to 0$ is different from the other frequency regimes. For better illustration, Fig. 2(b) shows $W(\omega)$ as a function of the electron velocity at three representative frequencies, namely 24.5 THz (within the first Reststrahlen band of BN) with $\varepsilon_L = 7.7 + 0.01i$ and $\varepsilon_z = -0.05 + 0.04i$, 42 THz (within the second Reststrahlen band) with $\varepsilon_L = -34.8 + 4.6i$ and $\varepsilon_z = 2.7 + 0.0005i$, and 35 THz (outside these two Reststrahlen bands) with $\varepsilon_L = 11.6 + 0.1i$ and $\varepsilon_z = 2.5 + 0.001i$. At 35 or 42 THz without $\varepsilon_z \to 0$, $W(\omega)$ monotonically increases with $v$ in Fig. 2(b). By contrast, at $\omega_0/2\pi = 24.5$ THz with $\varepsilon_z \to 0$, $W(\omega_0)$ first increases with $v$ if $v < v_A$, becomes insensitive to the variation of $v$ if $v \in [v_A, v_B]$, then decreases with $v$ if $v \in [v_B, v_C]$, and increases again with $v$ if $v > v_C$, where $v_A = 4.5 \times 10^{-4}c$, $v_B = 2.6 \times 10^{-1}c$, and $v_C = 0.9c$. Remarkably, Fig. 2(b) also shows that the radiation intensity at the frequency with $\varepsilon_z \to 0$ can be 2 orders of magnitude larger than that at the frequency without $\varepsilon_z \to 0$ if $v/c < 10^{-1}$. Moreover, the values of $W(\omega_0)$ are the same, if the electron velocity is equal to $v_A$, $v_B$, or $v_D$, where $v_D = 0.999c$. For these velocities, the corresponding kinetic energy varies from $E_{k,A} = 51.7$ meV, $E_{k,B} = 18.2$ keV to $E_{k,D} = 10.9$ MeV. This exotic feature at 24.5 THz in Figs. 2(a) and 2(b) indicates that the transition radiation from low-energy particles can achieve the same radiation intensity as that from high-energy particles.

Upon close inspection, we find that the analytical limit of total energy spectrum $W(\omega)$ is essentially irrelevant to $v$, under the conditions of $\varepsilon_z(\omega) \to 0$, $\omega d/c \ll \sqrt{|\varepsilon_z|/\varepsilon_\perp}$, and $\omega d/c \ll v/c \ll 1$; see Supplemental Material, Sec. S5, as well as Figs. S2 and S3 [54]. Moreover, the limit of $W(\omega)$ is almost proportional to $1/|\varepsilon_z|^2$ in Fig. S3. This mathematically explains the emergence of low-velocity-favored transition radiation from an ultrathin epsilon-near-zero slab, which is featured with a plateau of largely enhanced radiation spectrum within the velocity range of $v \in [v_A, v_B]$ in Fig. 2(b). Since the condition of $\omega d/c \ll v/c \ll 1$ is dependent on $d$, this velocity range within which the low-velocity-favored transition radiation could occur is sensitive to $d$ in Fig. S2.

According to Figs. 2(a) and 2(b), the photon extraction efficiency is further obtained as $\eta(\omega) = |W(\omega)|/\hbar \omega$; $E_k$, the rest mass, and $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor. Figure 2(c) shows the photon extraction efficiency as a function of frequency and velocity. From Fig. 2(c), if $v/c < 10^{-2}$, the maximum value of $\eta(\omega)$ always appears near the frequency with $\varepsilon_z \to 0$. Upon close inspection, Fig. 2(d) shows $\eta(\omega)$ as a function of velocity at three representative frequencies. When $v/c < 10^{-2}$, the photon extraction efficiency at 24.5 THz is nearly 3 orders of magnitude greater than those at 42 and 35 THz. Moreover, at $\omega_0/2\pi = 24.5$ THz, the value of $\eta(\omega_0)$ at point A with $v_A/c = 4.5 \times 10^{-4}$ is 8 orders of magnitude higher than that at point D with $v_D/c = 0.999$ in Fig. 2(d), although the values of $W(\omega_0)$ at these two points are the same in Fig. 2(b).

Besides the unique frequency spectral features in Fig. 2, this low-velocity-favored transition radiation also has exotic angular spectral features in Fig. 3. Figures 3(a)–3(d) illustrate the angular spectral energy density $U(\omega, \theta)$ as a function of velocity and radiation angle $\theta$. At the frequency with $\varepsilon_z \to 0$, the dependence of $U(\omega, \theta)$ on $v$ is not monotonical but rather complex in Figs. 3(a) and 3(b). Moreover, if the electron velocity is small (e.g., $v/c < 0.3$), the maximum of $U(\omega, \theta)$ still appears at a relatively large radiation angle ($>80^\circ$) with a relatively large angular width ($>50^\circ$) in Figs. 3(a) and 3(b). This feature for low-energy electrons is different from that for high-energy electrons. Generally, if $v \to c$ or $\gamma \gg 1$, the maximum of $U(\omega, \theta)$ would appear at $\theta \to 0^\circ$ with a very narrow angular width ($<0.1^\circ$) [9]. Since the resonance transition radiation is able to simultaneously improve the directivity and intensity of light emission [2], the relatively poor directivity of low-velocity-favored transition radiation might be overcome through the formation of resonance transition radiation by letting the moving electron interact with an epsilon-near-zero-material-based photonic crystal.
Besides, the angular spectral feature of transition radiation at the frequency with $\varepsilon_z \to 0$ in Figs. 3(a) and 3(b) is entirely different from those at frequencies without $\varepsilon_z \to 0$ in Figs. 3(c) and 3(d). At the frequency without $\varepsilon_z \to 0$, $U(\omega, \theta)$ in Figs. 3(c) and 3(d) monotonically increases with $\nu$. Meanwhile, the maximum of $U(\omega, \theta)$ starts to appear at a relatively small radiation angle ($< 45^\circ$) if $v/c > 0.3$ in Figs. 3(c) and 3(d).

To confirm the unique angular feature above, Figs. 3(e) and 3(f) show the field distribution of the excited waves at the frequency with $\varepsilon_z \to 0$. Remarkably, the field strength with $v/c = 0.001$ in Fig. 3(f) is comparable to that with $v/c = 0.999$ in Fig. 3(e). However, the excited waves mainly propagate to the directions almost parallel to the interface (with $\theta \to 90^\circ$) if $v/c = 0.001$ in Fig. 3(f), which is similar to the dipolar radiation induced by a dipole oscillating in a direction vertical to the epsilon-near-zero slab, while most of the excited waves would propagate to the direction almost perpendicularly to the interface (with $\theta \to 0^\circ$) if $v/c = 0.999$ in Fig. 3(e). For comparison, we show the excited waves at the frequency without $\varepsilon_z \to 0$ [e.g., in Figs. 3(g) and 3(h), and in Fig. S6 [54]]. By contrast, the field strength with $v/c = 0.001$ in Fig. 3(h) is much weaker than that with $v/c = 0.999$ in Fig. 3(g).

Since the low-velocity-favored transition radiation mainly occurs near the frequency with $\varepsilon_z \to 0$, its origin is closely related to the excitation of Ferrell-Berreman modes [4,40,41]. Essentially, the underlying mechanism for the transition radiation of Ferrell-Berreman modes is that the bulk plasmons provide a unique route to extend the electron-interface interaction time, then create light emission far beyond the conventional formation time historically defined for free-electron radiation, and thus help to greatly enhance the radiation intensity [4]. In other words, the emergence of a long tail of bulk plasmons following the electron’s trajectory deep into the epsilon-near-zero material mixes surface and bulk effect, and it provides a sustained channel for electron-interface interaction [4]. In addition, the consideration of the nonlocal response of epsilon-near-zero materials and the excitation of longitudinal waves has a minor influence on the angular spectral energy density and the radiation spectrum of excited propagating waves [4]. On the other hand, the low-velocity-favored transition radiation can occur if a moving electron penetrates through an epsilon-near-zero slab in Figs. 2 and 3 but would not appear if the electron crosses a single interface between an epsilon-near-zero material and free space in Fig. S1, despite the excitation of Ferrell-Berreman modes in both scenarios. In this way, the revealed phenomenon of low-velocity-favored transition radiation, including the plateau and the dip in Fig. 2(b), could be ascribed to the interference between the excited Ferrell-Berreman modes, instead of merely the excitation of Ferrell-Berreman modes.

The Ferrell-Berreman mode itself has been extensively studied, including that in anisotropic systems [41,65–67]. The transition radiation of Ferrell-Berreman modes has also been extensively discussed but is focused on the isotropic materials (e.g., a metal slab) [4,40,46–49]. However, the study of transition radiation of Ferrell-Berreman modes in
The phenomenon of the low-velocity-favored transition radiation in various epsilon-near-zero materials. (a) Radiation spectrum of transition radiation from a uniaxial epsilon slab with the consideration of material loss. The radiation peak near the frequency with \( \epsilon_z \to 0 \) is related to the excitation of Ferrell-Berrema (FB) modes, while there is no radiation peak near the frequency with \( \epsilon_{\perp} \to 0 \). (b) Radiation spectrum of transition radiation from uniaxial materials with \( |\epsilon_z| \to 0 \); the structural setup is the same as Fig. 2(a), except for the permittivity of region 2. For conceptual illustration, the material loss is neglected in (b).

FIG. 4. Existence of the low-velocity-favored transition radiation in various epsilon-near-zero materials. (a) Radiation spectrum of transition radiation from a uniaxial epsilon slab with the consideration of material loss. The radiation peak near the frequency with \( \epsilon_z \to 0 \) is related to the excitation of Ferrell-Berrema (FB) modes, while there is no radiation peak near the frequency with \( \epsilon_{\perp} \to 0 \). (b) Radiation spectrum of transition radiation from uniaxial materials with \( |\epsilon_z| \to 0 \); the structural setup is the same as Fig. 2(a), except for the permittivity of region 2. For conceptual illustration, the material loss is neglected in (b).

To address this issue, Fig. 4(a) replots the radiation spectrum of transition radiation from BN in Fig. 2(a) under three fixed velocities. For BN, we have \( \epsilon_{\perp} \to 0 \) near 48.2 THz (namely \( \epsilon_{\perp} = 0.02 + 0.08i \) and \( \epsilon_z = 2.8 + 0.0003i \)), in addition to \( \epsilon_z \to 0 \) near 24.5 THz. A radiation peak, which is a characteristic signature of the Ferrell-Berrema mode, always shows up near 24.5 THz but does not emerge near 48.2 THz in Fig. 4(a). Hence, \( \epsilon_z \to 0 \), instead of \( \epsilon_{\perp} \to 0 \), plays a crucial role in the excitation of Ferrell-Berrema modes. Moreover, Fig. 4(b) shows the transition radiation from various uniaxial epsilon-near-zero materials with \( |\epsilon_z| \to 0 \), where the material loss is neglected and the other structural setup is the same as that in Fig. 2(a). The phenomenon of the low-velocity-favored transition radiation always appears, no matter \( \text{Re}(\epsilon_z) > 0 \) or \( \text{Re}(\epsilon_z) < 0 \). Meanwhile, the appearance of this phenomenon is relatively insensitive to the values of \( \epsilon_{\perp} \) in Fig. 4(b). Figure 4(b) also indicates that the occurrence of low-velocity-favored transition radiation does not necessarily require the existence of high-\( \epsilon \) modes in epsilon-near-zero materials (Fig. S5 [54]), since it is not caused by Cherenkov radiation of high-\( \epsilon \) modes (Fig. S9) (e.g., that inside a hyperbolic material [28–32]). From Figs. 4(a) and 4(b), we then conclude that \( |\epsilon_z| \to 0 \) plays a determinant role in the creation of the low-velocity-favored transition radiation. Therefore, it is better to exploit \( |\epsilon_z| \to 0 \), instead of \( |\epsilon_{\perp}| \to 0 \), for the design of novel light sources based on low-energy electrons.

In conclusion, we have demonstrated a feasible route to achieve the low-velocity-favored transition radiation by exploiting the Ferrell-Berrema mode in epsilon-near-zero materials. Such an exotic phenomenon of free-electron radiation can simultaneously achieve strong emission intensity and high photon extraction efficiency readily from low-energy electrons. Because of the abundance of epsilon-near-zero materials in nature or through judicious nanofabrication [68–73], our finding of low-velocity-favored transition radiation can apply to a broad range of frequencies, e.g., microwave, terahertz, midinfrared, visible, and ultraviolet. Therefore, our finding not only enriches the physics of free-electron radiation but also broadens potential applications of Ferrell-Berrema modes, especially for the design of integrated free-electron light sources that are highly efficient and tunable.

X. L. acknowledges the support partly from the National Natural Science Fund for Excellent Young Scientists Fund Program (Overseas) of China, the National Natural Science Foundation of China under Grant No. 62175212, Zhejiang Provincial Natural Science Fund Key Project under Grant No. LZZ3F050003, and the Fundamental Research Funds for the Central Universities (2021FZZX001-19). H. C. acknowledges the support from the Key Research and Development Program of the Ministry of Science and Technology under Grants No. 2022YFA1404704, No. 2022YFA1404902, and No. 2022YFA1405200, the National Natural Science Foundation of China under Grants No. 11961141010 and No. 61975176. J. C. acknowledges the support from the Key Research and Development Program of the Ministry of Science and Technology under Grants No. 830/19. Y. Y. acknowledges the support from the Key Research and Development Program of the Ministry of Science and Technology under Grants No. 830/19. Y. Y. acknowledges the support from the start-up fund of the University of Hong Kong and the National Natural Science Foundation of China Excellent Young Scientists Fund (HKU 12222417). B. Z. acknowledges the support from Singapore National Research Foundation Competitive Research Program No. NRF-CRP23-2019-0007.


