Supplementary Information
Spin-Spacetime Censorship: a gedanken experiment of quantum information in gravitational settings

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Section 1: Symmetry properties of the spacetime metric and measurements of all its components

In this section we discuss whether the spin value (up or down), rather than its axis only, has any effect on spacetime. Up to now we have mostly considered $g_{00}(x^\mu)$, which is often the most significant component of the metric, but to answer this question we shall now study all the 16 $g_{\alpha\beta}(x^\mu)$ terms, and analyze their symmetry properties.

Let us first discuss parity and time-reversal transformations. The parity transformation defined by $P(x^0,x^1,x^2,x^3)\rightarrow(x^0,-x^1,-x^2,-x^3)$ and the time-reversal transformation defined by $T(x^0, x^1, x^2, x^3) \to (-x^0, x^1, x^2, x^3)$ flip the sign of the spin. However, due to the tensor properties of the spacetime metric $g_{\alpha\beta}(x^\mu)$, both g_{00} and all of the g_{ij} $(1 \le i, j \le 3)$ remain invariant under both parity and time-reversal transformations, i.e., $g_{00}(x^\mu) = g_{00}(P(x^\mu)) = g_{00}(T(x^\mu))$ and $g_{ij}(x^\mu) = g_{ij}(P(x^\mu)) = g_{ij}(T(x^\mu))$. Since the spin flips its sign under these transformations, it follows that g_{00} and g_{ij} $(1 \le i, j \le 3)$ are independent with respect to the spin sign.

In contrast, the six g_{i0}, g_{0i} components flip their signs under parity and time-reversal transformations: $g_{i0}(x^\mu) = -g_{i0}(P(x^\mu)) = -g_{i0}(T(x^\mu)) = g_{0i}(x^\mu) = -g_{0i}(P(x^\mu)) = -g_{0i}(T(x^\mu))$. Since the spin also flips its sign under these transformations, the six g_{i0}, g_{0i} components are either (#1) correlated to the spin or (#2) nonexistent, i.e., equal to zero (if, in addition, g_{00}, g_{ij} are time-independent, then the spacetime would be called static). If g_{i0}, g_{0i} are nonzero and correlated to the spin, as in option (#1), then there should exist a stronger spin censorship mechanism preventing not just the detection of the spin axis (as in the main text) but also the detection of the spin direction. In the next paragraphs we prove that in order to prevent a paradox, the measurable spacetime and all components of $g_{\alpha\beta}(x^\mu)$ are both spherically symmetric and static, with g_{i0}, g_{0i} being zero, i.e., option (#2).

To continue, we show that all 16 terms of $g_{\alpha\beta}(x^\mu)$ can be measured with clocks, each moving along a different timelike trajectory in spacetime. Denote the trajectory of the clock as $x^\mu(t)$. The proper-time interval $d\tau$ along a short timelike 4-vector interval dx^μ of the trajectory $x^\mu(t)$ is given by $c^2 d\tau^2 = g_{\alpha\beta}(x^\mu(t)) dx^\alpha dx^\beta$. Thus, by using several variations of the clock's trajectory, one can determine *all* of the components of the spacetime metric $g_{\alpha\beta}(x^{\mu}(t))$. This is true, because $g_{\alpha\beta} = g_{\beta\alpha}$ and also because tensors of the form $dy^{\alpha}dy^{\beta}$ (where dy^{α} are time-like 4-vectors) span the space of (4×4) symmetric matrices (due to symmetry properties of the spacetime, one can greatly reduce the number of necessary measurements).

The ability to use clocks for measuring all the components of the spacetime metric $g_{\alpha\beta}(x^{\mu}(t))$ suggests a generalized gedanken experiment that follows the same lines of the experiment in the main text, and implies that there exists a generalized spin spacetime censorship: It should be impossible to infer the spin axis and direction (of a spin-½ particle) from measuring any component of $g_{\alpha\beta}(x^{\mu}(t))$ and from any measurement of proper-time intervals $c^2 d\tau^2 = g_{\alpha\beta}(x^{\mu}(t))dx^{\alpha}dx^{\beta}$. Interestingly, such a generalized spin spacetime censorship principle imposes additional important constraints on the measurable values of the spacetime metric $g_{\alpha\beta}$ around a spin-½ particle. The measurable tensor has to be spherically symmetric, and static with the six components g_{i0}, g_{0i} being zero. These constraints and other additional symmetry requirements will be presented below.

We introduce two additional symmetry properties of the spacetime metric that are associated with the spin: (1) invariance of the metric tensor with respect to continuous time translations; (2) invariance of the metric tensor with respect to continuous rotations around the spin axis. A spacetime endowed with these two symmetries is both stationary and cylindrically symmetric. With a proper choice of spherical coordinate system (t, r, θ, ϕ), it is well known that the most general possible form of this metric is given by[1],

$$c^2 d\tau^2 = e^{2\lambda(r,\theta)} c^2 dt^2 - e^{2\eta(r,\theta)} dr^2 - e^{2\psi(r,\theta)} r^2 d\theta^2 - e^{2\tilde{\psi}(r,\theta)} r^2 \sin(\theta)^2 \left(d\phi - \omega(r,\theta)dt\right)^2, \quad (1)$$

where θ is the polar angle measured with respect to the spin axis and ϕ is the azimuthal angle (describing rotations around the axis of the spin). We can then measure the components of this spacetime metric, with clocks moving along a specific timelike 4-vector in the spacetime. Choosing for the clocks stationary 4-vector paths of the form $(t_0, r_0, \theta_0, \phi_0) + (dt, 0, 0, 0)$, we see that spin spacetime censorship is maintained if and only if $\lambda(r,\theta) = \lambda(r)$ - i.e., λ is only a function of r (independent of θ). Furthermore, choosing radial paths for the clocks, $(t_0, r_0, \theta_0, \phi_0) + (dt, u_r dr, 0, 0)$ with $|u_r| < c$, we see that spin spacetime censorship is maintained if and only if $\eta(r,\theta) = \eta(r)$. Finally, choosing spherical tangent trajectories of the form $(t_0, r_0, \theta_0, \phi_0) + (dt, 0, u_\theta dt, u_\phi dt)$ with $r_0^2 u_\theta^2 + r_0^2 \sin(\theta_0)^2 u_\phi^2 = u_{\text{clock}}^2 < c^2$ we see that in order to maintain the spin spacetime censorship, the proper-time $d\tau$ must be a function of just two parameters: the radial coordinate r and the speed u_{clock}. It follows that we must have $\tilde{\psi}(r,\theta) = \psi(r,\theta) = \psi(r)$ and $\omega(r,\theta) = 0$. Thus, spin-spacetime censorship seems to allow only spacetime metrics equivalent, upon measurement, to $c^2 d\tau^2 = e^{2\lambda(r)} c^2 dt^2 - e^{2\eta(r)} dr^2 - e^{2\psi(r)} \left(r^2 d\theta^2 + r^2 \sin(\theta)^2 d\phi^2)\right)$, which implies spherical symmetry (for a precise definition see e.g., [2,3]). This spherical symmetry of the spacetime guarantees that the spin's axis and the spin's direction cannot be inferred with clocks regardless of their trajectories.

To summarize, this section showed that generalizing our gedanken experiment enables using a clock to measure all the components of $g_{\alpha\beta}(x^\mu)$. In principle, the terms g_{i0}, g_{0i} should enable measuring the direction of the spin, and not only its axis, as in the main text. Therefore, a classical way to prevent a paradox in our gedanken experiment is by determining that the measurable spacetime and all the $g_{\alpha\beta}(x^{\mu})$ components are spherically symmetric and static, with g_{i0}, g_{0i} being zero. As explained in the main text, some quantum approaches could potentially bypass a small classical deviation from spherical symmetry.

Section 2: A related gedanken experiment with spatial degrees of freedom instead of spin

This section, which continues the theme of combining concepts from general relativity and quantum information, describes a related gedanken experiment. In this additional gedanken experiment, the entangled variables of Alice and Bob are their particles' positions instead of spins. This gedanken experiment is a modified version of the one presented in[4]. It could be utilized to present some well-known conceptual problems with the semi-classical model (according to which it is assumed that the spacetime curvature is proportional to the expectation value of the stress-energy tensor[4,5]). While the semi-classical approximation creates a paradox with the gedanken experiment presented in this section, we will show that a simple quantization of linearized gravity leads to physically sound results (no paradox), namely that relativistic causality is maintained. We also discuss the difference between the two experiments, i.e. why the original gedanken experiment from the main text is not resolved (i.e., causality and no cloning are not maintained) by such a quantization of linearized gravity (in which each of the particles' ket states couples to a different spacetime ket state).

To analyze this gedanken experiment, consider an entangled state of the form

$$\begin{aligned}|\psi\rangle &= \frac{1}{\sqrt{2}}\left(\left|\mathbf{o}_B+\mathbf{R}_{\mathbf{p}}\right\rangle_B \otimes \left|\mathbf{o}_A+\mathbf{R}_{\mathbf{p}}\right\rangle_A + \left|\mathbf{o}_B-\mathbf{R}_{\mathbf{p}}\right\rangle_B \otimes \left|\mathbf{o}_A-\mathbf{R}_{\mathbf{p}}\right\rangle_A\right) = \\ &\equiv \frac{1}{\sqrt{2}}\left(\left|+\mathbf{B}\right\rangle_B \otimes \left|+\mathbf{A}\right\rangle_A + \left|-\mathbf{B}\right\rangle_B \otimes \left|-\mathbf{A}\right\rangle_A\right), \qquad (2)\end{aligned}$$

where $\left|\mathbf{o}_A + \mathbf{R}_\mathbf{p}\right\rangle_A \equiv \left|+\mathbf{A}\right\rangle_A$ and $\left|\mathbf{o}_A - \mathbf{R}_\mathbf{p}\right\rangle_A \equiv \left|-\mathbf{A}\right\rangle_A$ denote two discrete ket state particle positions in Alice's lab (and similarly, $\left|\mathbf{o}_B + \mathbf{B}\right\rangle_B \equiv \left|+\mathbf{B}\right\rangle_B$, $\left|\mathbf{o}_B - \mathbf{B}\right\rangle_B \equiv \left|-\mathbf{B}\right\rangle_B$ denote discrete ket state particle positions in Bob's lab).

In an attempt to communicate with Bob, Alice can measure her particle using any basis she wants to choose. Then, Bob can use clocks to measure time-dilation effects in an attempt to decipher Alice's choice of measurement basis. Using the linear approximation of gravity we find that, $g_{00,+\mathbf{B}}(\mathbf{r}) \approx \eta_{00} + h_{00}(\mathbf{r}) = 1 + \frac{2G}{c^4}\left(mc^2 \frac{1}{|\mathbf{r}-\mathbf{B}|}\right)$ if Bob's particle is in $\left|+\mathbf{B}\right\rangle_B$ and $g_{00,-\mathbf{B}}(\mathbf{r}) \approx \eta_{00} + h_{00}(\mathbf{r}) = 1 + \frac{2G}{c^4}\left(mc^2 \frac{1}{|\mathbf{r}+\mathbf{B}|}\right)$ if his particle is in $\left|-\mathbf{B}\right\rangle_B$. Thus, the combined state of the system (taking into account Bob's time dilation measurements) is described by

$$\left|\psi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|g_{00,+\mathbf{B}}(\mathbf{r})\right\rangle \otimes \left|+\mathbf{B}\right\rangle_B \otimes \left|+\mathbf{A}\right\rangle_A + \left|g_{00,-\mathbf{B}}(\mathbf{r})\right\rangle \otimes \left|-\mathbf{B}\right\rangle_B \otimes \left|-\mathbf{A}\right\rangle_A\right). \tag{3}$$

Now let us consider the effect of Alice's choice of measurement basis. Alice can choose any basis of the form $\left\{\alpha\left|+\mathbf{A}\right\rangle_A + \beta\left|-\mathbf{A}\right\rangle_A,\ \beta^*\left|+\mathbf{A}\right\rangle_A - \alpha^*\left|-\mathbf{A}\right\rangle_A\right\}$. The effect of her measurement on Bob's particle is described by tracing out the density matrix $\rho = \left|\psi\right\rangle\left\langle\psi\right|$ with respect to her measurement basis. Noting that

$$\left(\alpha^*\left\langle+\mathbf{A}\right|_A + \beta^*\left\langle-\mathbf{A}\right|_A\right)\left|\psi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|g_{00,+\mathbf{B}}(\mathbf{r})\right\rangle \otimes \alpha^*\left|+\mathbf{B}\right\rangle_B + \left|g_{00,-\mathbf{B}}(\mathbf{r})\right\rangle \otimes \beta^*\left|-\mathbf{B}\right\rangle_B\right)$$

$$\left(\beta\left\langle+\mathbf{A}\right|_A - \alpha\left\langle-\mathbf{A}\right|_A\right)\left|\psi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|g_{00,+\mathbf{B}}(\mathbf{r})\right\rangle \otimes \beta\left|+\mathbf{B}\right\rangle_B - \left|g_{00,-\mathbf{B}}(\mathbf{r})\right\rangle \otimes \alpha\left|-\mathbf{B}\right\rangle_B\right) \tag{4}$$

we obtain a reduced density matrix of the form

$$\begin{aligned}\rho_B &= \mathrm{Tr}_A\rho = {}_A\left\langle+\mathbf{A}\right|\rho\left|+\mathbf{A}\right\rangle_A + {}_A\left\langle-\mathbf{A}\right|\rho\left|-\mathbf{A}\right\rangle_A = \\ &= \frac{1}{2}\left[\left(\left|g_{00,+\mathbf{B}}(\mathbf{r})\right\rangle \otimes \alpha^*\left|+\mathbf{B}\right\rangle_B + \left|g_{00,-\mathbf{B}}(\mathbf{r})\right\rangle \otimes \beta^*\left|-\mathbf{B}\right\rangle_B\right)\left(\left\langle g_{00,+\mathbf{B}}(\mathbf{r})\right| \otimes \alpha\ {}_B\left\langle+\mathbf{B}\right| + \left\langle g_{00,-\mathbf{B}}(\mathbf{r})\right| \otimes \beta\ {}_B\left\langle-\mathbf{B}\right|\right)\right] \\ &+ \frac{1}{2}\left[\left(\left|g_{00,+\mathbf{B}}(\mathbf{r})\right\rangle \otimes \beta\left|+\mathbf{B}\right\rangle_B - \left|g_{00,-\mathbf{B}}(\mathbf{r})\right\rangle \otimes \alpha\left|-\mathbf{B}\right\rangle_B\right)\left(\left\langle g_{00,+\mathbf{B}}(\mathbf{r})\right| \otimes \beta^*\ {}_B\left\langle+\mathbf{B}\right| - \left\langle g_{00,-\mathbf{B}}(\mathbf{r})\right| \otimes \alpha^*\ {}_B\left\langle-\mathbf{B}\right|\right)\right].\end{aligned} \tag{5}$$

We then trace out the spatial degrees of freedom $\left\{ \left|+\mathbf{B}\right\rangle_B, \left|-\mathbf{B}\right\rangle_B \right\}$ by calculating

$$\mathrm{Tr}_B \rho_B = \mathrm{Tr}_{\{|+\mathbf{B}\rangle, |-\mathbf{B}\rangle\}} \rho_B = {}_B\left\langle +\mathbf{B} \right| \rho_B \left| +\mathbf{B} \right\rangle_B + {}_B\left\langle -\mathbf{B} \right| \rho_B \left| -\mathbf{B} \right\rangle_B . \tag{6}$$

We note that

$$\left\langle +\mathbf{B} \right|_B \rho_B \left| +\mathbf{B} \right\rangle_B = \frac{1}{2}\left[\left(\alpha^* \left| g_{00,+\mathbf{B}}(\mathbf{r}) \right\rangle \right) \left(\left\langle g_{00,+\mathbf{B}}(\mathbf{r}) \right| \alpha \right) \right] + \frac{1}{2}\left[\left(\beta \left| g_{00,+\mathbf{B}}(\mathbf{r}) \right\rangle \right) \left(\left\langle g_{00,+\mathbf{B}}(\mathbf{r}) \right| \beta^* \right) \right]$$

$$\left\langle -\mathbf{B} \right|_B \rho_B \left| -\mathbf{B} \right\rangle_B = \frac{1}{2}\left[\left(\beta^* \left| g_{00,-\mathbf{B}}(\mathbf{r}) \right\rangle \right) \left(\left\langle g_{00,-\mathbf{B}}(\mathbf{r}) \right| \beta \right) \right] + \frac{1}{2}\left[\left(\alpha \left| g_{00,-\mathbf{B}}(\mathbf{r}) \right\rangle \right) \left(\left\langle g_{00,-\mathbf{B}}(\mathbf{r}) \right| \alpha^* \right) \right], \tag{7}$$

and hence we obtain the reduced density matrix that describes Bob's clocks:

$$\begin{aligned} &\rho_{clocks} = \mathrm{Tr}_B \left\langle \rho_B \right\rangle = \\ &\frac{1}{2}\left[\left(|\alpha|^2 + |\beta|^2 \right) \left| g_{00,+\mathbf{B}}(\mathbf{r}) \right\rangle \otimes \left\langle g_{00,+\mathbf{B}}(\mathbf{r}) \right| \right] + \frac{1}{2}\left[\left(|\beta|^2 + |\alpha|^2 \right) \left| g_{00,-\mathbf{B}}(\mathbf{r}) \right\rangle \otimes \left\langle g_{00,-\mathbf{B}}(\mathbf{r}) \right| \right] = \\ &= \frac{1}{2} \left| g_{00,+\mathbf{B}}(\mathbf{r}) \right\rangle \otimes \left\langle g_{00,+\mathbf{B}}(\mathbf{r}) \right| + \frac{1}{2} \left| g_{00,-\mathbf{B}}(\mathbf{r}) \right\rangle \otimes \left\langle g_{00,-\mathbf{B}}(\mathbf{r}) \right|, \end{aligned} \tag{8}$$

which is maximally mixed (the probabilities of detecting the original locations $\left| \mathbf{o}_B + \mathbf{B} \right\rangle_B = \left| +\mathbf{B} \right\rangle_B$, $\left| \mathbf{o}_B - \mathbf{B} \right\rangle_B = \left| -\mathbf{B} \right\rangle_B$ are both equal to ½). This matrix is completely independent of any basis used in the description of Alice's measurement process - precisely what we need to ensure that the "no signaling" principle is obeyed.

Going back to our "spin-based" gedanken experiment (from the main text), we can ask: Would the quantization of linearized gravity maintain relativistic causality, as it did for the gedanken experiment presented here? Would there be equivalent consequences to the two gedanken experiments?

We show that the answer to both questions is **no**. The spin offers unique consequences.

While it seems at first that the above spatial gedanken experiment is similar to the spin gedanken experiment, they are inherently different: The algebra of spin addition is different from that of spatial coordinates. The superposition of spin states along the $\hat{\mathbf{x}}$ axis can end up in a spin oriented along the $\hat{\mathbf{y}}$ axis, which cannot occur with spatial coordinates. In practical terms, it seems that **time dilation measurements can localize a particle but they cannot determine its spin orientation**. So it is the richness of the spin algebra, and the specific way in which it couples to the spacetime, that leads to interesting and important consequences, which cannot be obtained with the spatial version of the gedanken experiment.

Section 3: A quantum description for the spacetime associated with a single spin-half particle and a quantum measurement with an ideal clock

In this section, we attempt to construct a quantum description of a single spin-half particle that includes both its spin and the surrounding spacetime. We begin by describing the quantum state of a stationary (i.e. completely delocalized) spin-half particle with a specific spin state $\left|\mathbf{S}^{\mu}\right\rangle$ that is coupled to the spacetime. This state is translation invariant, and therefore translating the particle's state and summing over all the possible translations, we obtain the following quantum representation:

$$\left|\mathbf{S}^{\mu}\right\rangle \otimes \int d^{3}\mathbf{x} \exp\left(-{}^{i}\!/_{\hbar} E_{0} t\right)\left|ST(\mathbf{x},\mathbf{S}^{\mu})\right\rangle \ , \tag{9}$$

where E_0 is the rest mass and $\left|ST(\mathbf{x},\mathbf{S}^{\mu})\right\rangle$ denotes the spacetime quantum ket vector associated with a spin $\mathbf{S}^{\mu}$ located at $\mathbf{x}$.

By boosting this stationary particle state, we obtain the state of a completely delocalized particle with momentum $\mathbf{p}$:

$$\left|\mathbf{S}^{\mu}(\mathbf{p})\right\rangle \otimes \int d^{3}\mathbf{x} \exp\left({}^{i}\!/_{\hbar}(\mathbf{p}\cdot\mathbf{x}-E_{\mathbf{p}}t)\right) B(\mathbf{p})\left|ST(\mathbf{x},\mathbf{S}^{\mu})\right\rangle, \tag{10}$$

where $\left|\mathbf{S}^{\mu}(\mathbf{p})\right\rangle = \left|\Lambda^{\mu}{}_{\mu'}(\boldsymbol{\beta}_{\mathbf{p}})\mathbf{S}^{\mu'}\right\rangle$ is the Dirac spinor associated with momentum $\mathbf{p}$, spin $\mathbf{S}^{\mu}$, boost spin transformation $\Lambda^{\mu}{}_{\mu'}(\boldsymbol{\beta}_{\mathbf{p}})$, velocity $\boldsymbol{\beta}_{\mathbf{p}} = \mathbf{u}_{\mathbf{p}} / c$, $(\mathbf{p} = (1-\boldsymbol{\beta}_{\mathbf{p}}{}^{2})^{-1/2} m_0 \mathbf{u}_{\mathbf{p}})$, and a boost operator $B(\mathbf{p})$ imbuing a particle at rest with momentum $\mathbf{p}$. It should be noted that $B(\mathbf{p})$ acts on the entire spacetime of the particle, applying to it a Lorenz transformation $L^{\mu}{}_{\mu'}(\boldsymbol{\beta}_{\mathbf{p}})$. The spacetime metric $\tilde{g}_{\alpha\beta}\left[x^{\nu}\right]$ associated with the boosted spacetime ket state $B(\mathbf{p})\left|ST(\mathbf{x},\mathbf{S}^{\mu})\right\rangle$ is just a Lorentz transformation $L^{\mu}{}_{\mu'}(\boldsymbol{\beta}_{\mathbf{p}})$ of the spacetime metric $g_{\alpha\beta}\left[x^{\nu};\left|ST(\mathbf{x},\mathbf{S}^{\mu})\right\rangle\right]$ associated with the state $\left|\text{Spacetime of spin } \mathbf{S}^{\mu} \text{ at } \mathbf{x}\right\rangle$ - i.e.,

$$\tilde{g}_{\alpha\beta}[x^{\nu}] = L_{\alpha}{}^{\alpha'}(\boldsymbol{\beta}_{\mathbf{p}}) L_{\beta}{}^{\beta'}(\boldsymbol{\beta}_{\mathbf{p}}) g_{\alpha'\beta'}\left[L^{\nu}{}_{\nu'}(\boldsymbol{\beta}_{\mathbf{p}}) x^{\nu'};\left|ST(\mathbf{x},\mathbf{S}^{\mu})\right\rangle\right], \tag{11}$$

Finally, we construct a general quantum state as the superposition of these boosted states, and thus it is formally described by,

$$\sum_{\mathbf{S}^{\mu}} \int d^3\mathbf{p}\, a\left(\mathbf{p},\mathbf{S}^{\mu}\right)\left|\mathbf{S}^{\mu}(\mathbf{p})\right\rangle \otimes \int d^3\mathbf{x}' \exp\left(\tfrac{i}{\hbar}(\mathbf{p}\cdot\mathbf{x} - E_{\mathbf{p}} t)\right) B(\mathbf{p})\left|ST(\mathbf{x},\mathbf{S}^{\mu})\right\rangle, \tag{12}$$

where, $a\left(\mathbf{p},\mathbf{S}^{\mu}\right)$ are the amplitudes associated with each momentum and spin state. It is important to note that even-though that each plane wave solution as in (10) is completely delocalized, we are particularly interested in the case in which their superposition is localized.

Using this approach, we now turn to analyze our gedanken experiment quantum mechanically. To do this, we shall calculate the possible values of the spacetime metric element g_{00} at an arbitrary spacetime point $(0,\mathbf{x}_0)$. We shall compute this with the aid of an ideal quantum clock located at time $t = 0$ at $\mathbf{x}_0$. An ideal clock could be a qubit that is used for tracking the time. It could be, for example, a two-level atomic system of the form $\left|\varphi_{\text{clock}}(t)\right\rangle = \left(\left|0\right\rangle + \exp(-i\omega_{\text{clock}} t)\left|1\right\rangle\right) \otimes \int d\mathbf{x}_c \varphi(t,\mathbf{x}_c - \mathbf{x}_0)\left|\mathbf{x}_c\right\rangle$, where $\varphi_c(t,\mathbf{x}_c - \mathbf{x}_0)$ is a wavepacket (located at the vicinity of $\mathbf{x}_0$) describing the position of the clock and $|0\rangle + \exp(-i\omega_{\text{clock}} t)|1\rangle$ is a time dependent qubit (realized, e.g., by two energy levels of the atom[6]). For simplicity, we shall now assume that the clock's mass and energy are small when compared to the mass and energy of the spin-half particle (but the following calculations can be generalized provided that the clock and the particle are separated by a sufficiently large distance). Furthermore, before we continue, let us note that the evolution of the clock's wave function under the influence of an external gravitational field is given by,

$$\begin{aligned}&|\varphi_{\text{clock}}(dt)\rangle = \\ &\quad = \int d^3\mathbf{x}_c \left(|0\rangle + \exp\left(-i\theta_{\text{clock}} dt \sqrt{g_{00}[0,\mathbf{x}_c]}\right)|1\rangle\right) \otimes \varphi_c(dt\sqrt{g_{00}[0,\mathbf{x}_c]}, \mathbf{x}_c - \mathbf{x}_0)|\mathbf{x}_c\rangle.\end{aligned} \tag{13}$$

Thus, by measuring the qubit's $|0\rangle + \exp\left(-i\theta_{\text{clock}} dt \sqrt{g_{00}[0,\mathbf{x}_0]}\right)|1\rangle$ state in the computational basis, we can find the possible values of the parameter $\sqrt{g_{00}[0,\mathbf{x}_0]}$. Now let us consider the case where the clock is influenced by a *quantum superposition of several gravitational fields* (induced by the wavefunction of the spin-½ particle $|\psi_{\text{particle}}\rangle$). This measurement is described by noting that the quantum state of the *combined system*, i.e., the electron and the clock's time pointer at time dt, is given by

$$\begin{aligned}&|\psi_{\text{particle and clock's pointer}}(dt)\rangle = \\ &= \sum_{\mathbf{S}^\mu} \int d^3\mathbf{p}\, a\left(\mathbf{p},\mathbf{S}^\mu\right)\left|\Lambda^\mu_{\ \mu'}(\mathbf{p})\mathbf{S}^{\mu'}\right\rangle \otimes \\ &\quad \otimes \int d^3\mathbf{x} \exp\left(\tfrac{i}{\hbar}(\mathbf{p}\cdot\mathbf{x} - E_{\mathbf{p}} dt)\right) B(\mathbf{p}) \left|ST(\mathbf{x},\mathbf{S}^\mu)\right\rangle \otimes \\ &\quad\quad \otimes \int d^3\mathbf{x}_c \varphi_c \left(dt\sqrt{L_0^{\ \alpha}(\boldsymbol{\beta}_\mathbf{p}) L_0^{\ \beta}(\boldsymbol{\beta}_\mathbf{p}) g_{\alpha\beta}\left[L^\nu_{\ \nu'}(\boldsymbol{\beta}_\mathbf{p})\cdot(0,\mathbf{x}_c - \mathbf{x}); |ST(\mathbf{x},\mathbf{S}^{\mu})\rangle\right]}, \mathbf{x}_c - \mathbf{x}_0\right)|\mathbf{x}_c\rangle \otimes \\ &\quad\quad\quad \otimes \left(|0\rangle + \exp\left(-i\theta_{\text{clock}} dt \sqrt{L_0^{\ \alpha}(\boldsymbol{\beta}_\mathbf{p}) L_0^{\ \beta}(\boldsymbol{\beta}_\mathbf{p}) g_{\alpha\beta}\left[L^\nu_{\ \nu'}(\boldsymbol{\beta}_\mathbf{p})\cdot(0,\mathbf{x}_c - \mathbf{x}); |ST(\mathbf{x},\mathbf{S}^{\mu})\rangle\right]}\right)|1\rangle\right),\end{aligned} \tag{14}$$

where, as explained above, $L_0^{\ \alpha}(\boldsymbol{\beta}_{\mathbf{p}})L_0^{\ \beta}(\boldsymbol{\beta}_{\mathbf{p}})g_{\alpha\beta}\left[L^{\nu}_{\ \nu'}(\boldsymbol{\beta}_{\mathbf{p}})\cdot\left(0,\mathbf{x}_c-\mathbf{x}\right);\left|ST\,(\mathbf{x},\mathbf{S}^{\mu})\right\rangle\right]$ is the time dilation effect (at $\mathbf{x}_c$) which is associated with the boosted state $B(\mathbf{p})\left|\text{Spacetime of spin }\mathbf{S}^{\mu}\text{ at }\mathbf{x}\right\rangle$.

Finally, to calculate the possible values of the clock's time dilation, we simply trace out all the degrees of freedom associated with the electron and the clock's position. This way we obtain the clock's density matrix:

$$\begin{aligned}
\rho_{clcok}(dt) &= \int d^3\mathbf{x}\sum_{\mathbf{S}^{\mu}}\int d^3\mathbf{q}\int d^3\mathbf{p}\,\overline{a}\left(\mathbf{q},\mathbf{S}^{\mu}\right)a\left(\mathbf{p},\mathbf{S}^{\mu}\right)\exp\left(\tfrac{i}{\hbar}(\mathbf{p}-\mathbf{q})\mathbf{x}-\tfrac{i}{\hbar}(E_{\mathbf{p}}-E_{\mathbf{q}})dt\right)\cdot \\
&\cdot\int d^3\mathbf{x}_c\varphi_c\left(dt\sqrt{L_0^{\ \alpha}(\boldsymbol{\beta}_{\mathbf{p}})L_0^{\ \beta}(\boldsymbol{\beta}_{\mathbf{p}})g_{\alpha\beta}\left[L^{\nu}_{\ \nu'}(\boldsymbol{\beta}_{\mathbf{p}})\cdot\left(0,\mathbf{x}_c-\mathbf{x}\right);\left|ST\,(\mathbf{x},\mathbf{S}^{\mu})\right\rangle\right]},\mathbf{x}_c-\mathbf{x}_0\right)\cdot \\
&\quad\cdot\varphi_c\left(dt\sqrt{L_0^{\ \alpha}(\boldsymbol{\beta}_{\mathbf{q}})L_0^{\ \beta}(\boldsymbol{\beta}_{\mathbf{q}})g_{\alpha\beta}\left[L^{\nu}_{\ \nu'}(\boldsymbol{\beta}_{\mathbf{p}})\cdot\left(0,\mathbf{x}_c-\mathbf{x}\right);\left|ST\,(\mathbf{x},\mathbf{S}^{\mu})\right\rangle\right]},\mathbf{x}_c-\mathbf{x}_0\right)^{*}\cdot \\
&\cdot\left(\left|0\right\rangle+\exp\left(-i\theta_{\text{clock}}dt\sqrt{L_0^{\ \alpha}(\boldsymbol{\beta}_{\mathbf{p}})L_0^{\ \beta}(\boldsymbol{\beta}_{\mathbf{p}})g_{\alpha\beta}\left[L^{\nu}_{\ \nu'}(\boldsymbol{\beta}_{\mathbf{p}})\cdot\left(0,\mathbf{x}_c-\mathbf{x}\right);\left|ST\,(\mathbf{x},\mathbf{S}^{\mu})\right\rangle\right]}\right)\left|1\right\rangle\right) \\
&\quad\left(\left\langle 0\right|+\exp\left(+i\theta_{\text{clock}}dt\sqrt{L_0^{\ \alpha}(\boldsymbol{\beta}_{\mathbf{q}})L_0^{\ \beta}(\boldsymbol{\beta}_{\mathbf{q}})g_{\alpha\beta}\left[L^{\nu}_{\ \nu'}(\boldsymbol{\beta}_{\mathbf{q}})\cdot\left(0,\mathbf{x}_c-\mathbf{x}\right);\left|ST\,(\mathbf{x},\mathbf{S}^{\mu})\right\rangle\right]}\right)\left\langle 1\right|\right).
\end{aligned}\tag{15}$$

In particular, we see that spin-spacetime censorship is maintained if the spacetime metric is spin-independent. Note that this is a sufficient, but not necessary, solution to the question – how relativistic causality and no-cloning are maintained in our gedanken experiment.

Elaborating on the possibility that the spacetime metric is spin-independent, we note that it imposes a very strong condition of spin censorship, as it implies that the spacetime associated with a single spin is spherically symmetric. Assuming that the metric is indeed spherically symmetric can reduce the number of possible spacetime metrics according to Birkhoff's generalized theorem[2]. For example, one could consider the spherically symmetric Schwarzschild metric centered at the localized spin. This choice is consistent with our previously described dust stress-energy tensors approach (classical approach #2). However, if the spacetime (around the spin-½) is indeed spherically symmetric, then one may also expect certain classical ramifications for the EMFE. For example, many particles having all their spins pointing in the same direction, and thus forming a very strong magnetic field, may still show no break of spherical symmetry of the time dilation effect around them. Therefore, this particular solution (maintaining causality and no cloning in our gedanken experiment) seems to lead to a classical modification of EMFE with implications on cosmological scales, unless it can somehow be shown that the collective spacetime effect of many particles is different from that of a single particle (see e.g., the paragraph regarding an additive auxiliary stress energy tensor in SI section 5).

A different pathway for achieving spin censorship based on the formalism here involves an idea related to weak measurements[7]: consider a situation where the clock's state has a very broad distribution around the expectation value of θ_{clock} from which it is practically impossible to infer the axis of the spin (the back-action of the pointer on the measured system could also contribute to the accumulated uncertainty). Applying weak measurement is quite plausible due to the minuscule coupling strength between the spin and the clock, leading to a shift in the clock pointer that may be much smaller than its quantum uncertainty (even if the clock is very precise). The less obvious characteristic of this approach is finding how the measurement strength increases with duration and with the number of spins/clocks.

Section 4: Spin spacetime censorship for photons

In this section, we present an analogue to the spin gedanken experiment that applies to massless particles with spin, such as photons with polarizations. Photons can be entangled through their polarizations, for example, entanglement of the vertical (V) and horizontal (H) polarization according to $\left(|HV\rangle-|VH\rangle\right)/\sqrt{2}$. To explain such a gedanken experiment, recall that according to the EMFE, photons that are linearly polarized induce (very) weak gravitational pp-waves[8] with (vertical-horizontal) polarization relative to the photon polarization state (thus linearly polarized photons at $\pm 45^0$ induce a polarized mode of gravitational pp-wave at $\pm 45^0$). It therefore follows that if Alice projects her photon to a specific linearly polarized state, then Bob can (in theory) measure the gravitational wave induced by his photon to determine the induced polarization state. From a classical point of view, by measuring gravity waves, Bob can determine whether his photon is linearly polarized in one of (0^0,90^0) linear polarization states or whether it is linearly polarized in one of the ($+45^0$,-45^0) states. Again, if Alice is sufficiently far, this clearly violates relativistic causality. Altogether, we expect there to be spin censorship principles for any spin, not only ½.

Section 5: Attempts to explain the gedanken experiment results with classical speculative approaches

Additive auxiliary tensor

One could attempt to realize a spin-spacetime censorship mechanism, by adding an auxiliary aspherical spin-dependent stress-energy tensor that would cancel the aspherical part of the Maxwell stress-energy tensor. With this additional term, the spin-spacetime censorship is realized by construction for each particle. Next, one can imagine what happens if there are many particles (each with its own additional auxiliary stress energy tensor). Naturally, one would assume that these additional auxiliary stress energy tensors add up together (yielding a total tensor which, in contrast to the Maxwell tensor, is at most linearly proportional to the number of particles). This approach seems rather ad hoc, assuming a yet unknown stress energy tensor that somehow accompanies the ordinary Maxwell stress energy tensor, but it might find justification from other perspectives.

Electric dipole

Other censorship mechanisms may seem appealing at first but prove to be flawed. For example, one might suggest that the looked-for electron electric dipole[9,10] can eliminate the aspherical parts of the stress-energy tensor created by the spin magnetic dipole. Indeed, the standard model predicts a non-zero electron electric dipole moment[9,10], whose value has not been found yet. However, the state-of-the-art upper bound on the electron's electric dipole moment found experimentally[11] is too small to compensate for an aspherical spacetime curvature created by the much larger electron dipole moment.

Electron rotation (classical spin)

Another candidate for a censorship mechanism is based on attributing a different internal rotation rate (classical "spin") to the electron, so it bends spacetime in an aspherical way that cancels out the effect of the electron magnetic dipole moment. However, one can show that such a rotation cannot compensate for the asphericity arising due to the spatial extent of the magnetic field, unless the electron is taken to have an extended mass/charge distribution. Such an approach would corroborate past attempts to treat the electron as a Kerr-Newman black hole that has a rotation rate and a spin consistent with each other. For example, Carter showed[12] that a constant classical angular momentum of $\hbar/2$ gives rise to magnetic moment similar to that of the electron spin (see also[13]). Trying now to alter the rotation rate to ensure a spherically symmetric spacetime curvature would harm this essential consistency. When exploring these models, it is worth noting that they have been shown to suffer from a naked singularity[14] and closed timelike curves[14].

However, a relatively new ghost-free approach to gravity, known as infinite derivative gravity[15,16], can overcome this problematic ring singularity. In such a non-local theory, the gedanken experiment as a whole, and entanglement in particular, would have to be carefully analyzed before conclusions can be made.

Section 6: Revisiting non-commutative spacetime geometry and other quantum approaches that seem to be challenged

In this section, we revisit the proposed censorship mechanism of non-commutative geometry of spacetime mentioned in the main text. We can introduce a generalization of our gedanken experiment, which seems to prove that this mechanism cannot maintain causality for all possible setups, of the gedanken experiment: The signaling protocol between Alice and Bob can be performed with many pairs of entangled particles (used simultaneously and measured by Alice with a single Stern-Gerlach device), while Bob measures each of his particles separately with *two clocks per particle* (so that the two clocks are positioned symmetrically at locations $\pm L\hat{\mathbf{n}}$ with respect to the spin-½ particle). Bob also locates his clocks at different $\hat{\mathbf{n}}$ orientations around each particle. Again, he can then compare the clocks times and determine that the clocks in the $\pm\hat{\mathbf{x}}$ orientation, are faster or slower than the clocks placed in the $\pm\hat{\mathbf{y}}$ axes orientation. Now since there is one clock per particle, Bob can separate his particles (sufficiently far apart) to prevent any possible interference between the different time measurements. This generalized gedanken experiment shows that mutual influence between different clocks cannot be given as a reason to resolve the paradox, which appears to challenge non-commutative geometry of spacetime as a censorship mechanism. Other possible approaches can still be *combined with a non-commutative geometrical model* in an attempt to maintain relativistic causality in our gedanken experiment.

Another approach for maintaining relativistic causality (in our gedanken experiment) could be based on coupling the wave equations of the electron to those of the gravitational potential: adding the gravitational potential (derived via the Poisson equation with the electron's mass density as a source) to the electron quantum wave equation (e.g., the Newton-Schrodinger equation and its relativistic generalizations). We did not consider this type of censorship candidates because they were shown, alongside with other nonlinear modifications of the Schrodinger equation, to allow signaling[17].

Section 7: The Kerr–Newman metric in classical gravity

This section recalls the aspherical solution of the EMFE for an electron with a spin - For a comprehensive discussion of this topic see[18]. An exact electro-vacuum solution can be expressed in the Boyer–Lindquist coordinates t, r, θ, ϕ ($x = \sqrt{r^2 + a^2} \sin\theta \cos\phi$, $y = \sqrt{r^2 + a^2} \sin\theta \sin\phi$, $z = r\cos\theta$) with the spacetime metric

$$g_{\mu\nu} = \begin{bmatrix} c^2(\Delta - a^2\sin^2\theta)/\rho^2 & 0 & 0 & -2ac\sin^2\theta(\Delta - r^2 - a^2)/\rho^2 \\ 0 & -\rho^2/\Delta & 0 & 0 \\ 0 & 0 & -\rho^2 & 0 \\ -2ac\sin^2\theta(\Delta - r^2 - a^2)/\rho^2 & 0 & 0 & \sin^2\theta(a^2\Delta\sin^2\theta - r^4 - 2r^2a^2 - a^4)/\rho^2 \end{bmatrix}, \quad (16)$$

where $a = \dfrac{\hbar}{2Mc}$, $\rho^2 = r^2 + a^2\cos^2\theta$, $\Delta = r^2 - r_s r + a^2 + r_Q^2$, $r_s = \dfrac{2Gm_e}{c^2}$, $r_Q^2 = \dfrac{e^2 G}{4\pi\epsilon_0 c^4}$, and the electromagnetic vector potential is given by[18,19] $A_\mu = \left(\dfrac{r\, r_Q}{\rho^2}, 0, 0, -\dfrac{c^2\, a\, r\, r_Q \sin^2\theta}{\rho^2\, G\, m_e} \right)$. It should be noted that for all known spin-1/2 particles the Kerr–Newman metric leads to a naked singularity and to closed time-like curves[20]. The radius of this ring singularity is $r_a \approx \dfrac{\hbar}{2mc}$, which is 1.93×10^{-13}[m] for an electron. To avoid this ring singularity and other UV problems, Biswas et al.[21] have suggested a higher derivative covariant generalization of general relativity, which could be applied to our gedanken experiment as well. Other researchers have suggested different models for spin-1/2 particles (see e.g.,[22]).

Another aspect of the Kerr-Newmann spacetime is that it predicts the electric dipole moment of the electron to be zero (the expectations for a nonzero electron dipole moment are related to QED). However, we know that higher, extremely tiny, but non-zero even-multipoles are expected to exist in Kerr-Newmann spacetime. No such multipoles have been observed so far in experiments. There is a rich discussion in the literature about suitable modifications and perhaps an internal structure[22-29] that could make these multipoles much smaller, so to become consistent with current experiments. In either case, the higher multipoles would not prevent the break of the spacetime spherical symmetry at the heart of our gedanken experiment. Consequently, our work show that even generalized version of the Kerr-Newmann spacetime cannot model our gedanken experiment without a causality paradox. It is intriguing that quantum information considerations have such consequences on a basic problem in (classical) general relativity.

Section 8: Perturbative approach with linearized quantum gravity

In this section, we examine the gedanken experiment from the perspective of linearized quantum gravity (two recent accounts appear in[30,31]), where gravitons emerge through second quantization of a linearized perturbation to the metric. The inherently relativistic dynamics of these second-quantized linearized gravitational fields may seem to automatically solve the issues of causality (and retardation) in a manner similar to the case of quantum electrodynamics (QED). That is, in the same way that QED prevents measuring the spin with photons, linearized quantum gravity could be expected to prevent measuring the spin with gravitons.

In this approach, we analyze the spin coupling to spacetime by considering a *linearized theory*, where basic concepts that are familiar from flat space, such as angular momentum and dipole moment, carry over to curved spacetime[32].These concepts are needed to model our gedanken experiment. Therefore, one would naturally raise the question of whether linearized quantum gravity could model our gedanken experiment with no causality paradox.

To avoid the paradox, a specific mechanism is needed in linearized quantum gravity: one that will prevent Bob from inferring the axis and direction of the spin with his clocks. However, a few difficulties are encountered.

(1) There seem to be some core differences between QED and linearized quantum gravity (see also Appendix section 1), related to the different ways these theories couple to spin. Within the former, the scattering amplitude typically depends on the spin of the fermions (i.e. before taking the customary average of initial spins and sum of the final spins). In contrast, within the latter, it is unclear how to model the scattering by a spin (e.g., would scattering off the gravitational potential created by the spin depend on the entire state of the spin, or only its axis? and if so then how?).

(2) The literature seems to mostly analyze scenarios where the sources of the fields are classical[30,31], while the non-commutativity of spin components in our gedanken experiment makes it inherently quantum. Therefore, a quantum theory of linearized gravity *with non-classical sources* is needed.

(3) It is possible to use the ADM formalism and the Wheeler-DeWitt equation in its linearized form to describe the way in which spin sources gravity. However, in Appendix section 4 we proved that this approach fails (unless we assume that spin and spacetime are completely decoupled).

(4) Even in the case of a theory that treats a non-classical source, it seems that for preserving the total angular momentum, simple diagrams would not suffice: The graviton has spin 2, while the electron has spin ½ and thus coupling them seems to invoke multiple mediators (both photons and gravitons). Such a multi-particle interaction seems to require nonlinear interactions (i.e. vertices involving an electron, a photon and a graviton), which go beyond the scope of linearized quantum gravity. Alternatively, we can use higher order diagrams (see Fig. S1a) that are, however, diverging (containing at least one loop). There are other

possibilities (e.g., Fig. S1b and its inverse) that include processes such as photon emission by the spin absorbing a graviton (could be envisioned as radiation from a freely-falling electron). However, as of now, such processes are at most speculative.

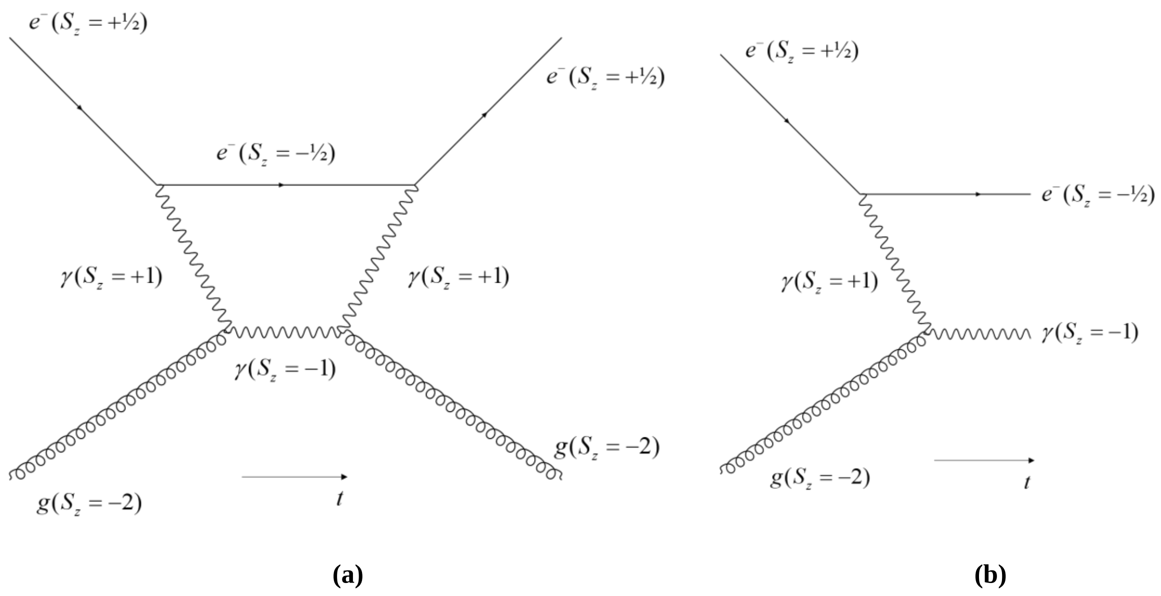

Figure S1: Feynman diagrams in linearized quantum gravity. (a) High order diagram containing one loop. The electron maintains its spin. (b) The electron flips its spin as a result of the interaction, in which it absorbs a graviton and emits a photon (possibly describing a spin-flip radiation process by fee-falling in the gravitational field).

To conclude, we are not aware of any satisfactory mechanism addressing our gedanken experiment within the current theory of linearized quantum gravity.

Section 9: Why a single spin-½ cannot be fully inferred with magnetic measurements

This section elaborates on the question of why the spin axis cannot be determined by measuring its induced magnetic field. It is the back-action of the quantum spin measurements on the spin axis that affects the spin and changes it (unless the measurements are performed along the spin axis). It seems plausible that a similar back-action would enable modeling our gedanken experiment without a contradiction with causality. We elaborate on this possibility in Appendix section 4 and SI section 8, showing mechanisms of back-action in quantum gravity that still result in a contradiction, unlike the case of quantum electrodynamics that we describe in this section.

A measuring apparatus (made of coils, magnets, etc.) that measures a spin does so by coupling to the spin – applying on it a magnetic field $\mathbf{B} = B\hat{\mathbf{n}} \neq 0$ even when spatially separated from the spin. The spin-½ state is influenced by this magnetic field. The magnetic moment μ of the spin and the magnetic field $\mathbf{B} = B\hat{\mathbf{n}}$ determine the spin's time dependent unitary evolution, which is given by the Schrödinger equation according to $i\hbar\partial_t |S(t)\rangle = \left(\frac{\mu B}{2}|+\hat{\mathbf{n}}\rangle\langle+\hat{\mathbf{n}}| - \frac{\mu B}{2}|-\hat{\mathbf{n}}\rangle\langle-\hat{\mathbf{n}}|\right)|S(t)\rangle$. The spin-½ evolution is thus given by $|S(t)\rangle = \gamma_{+\hat{\mathbf{n}}} \exp\left(i\frac{\mu B}{2\hbar}t\right)|+\hat{\mathbf{n}}\rangle + \gamma_{-\mathbf{n}} \exp\left(-i\frac{\mu B}{2\hbar}t\right)|-\hat{\mathbf{n}}\rangle$, where $\gamma_{\pm\hat{\mathbf{n}}}$ are constants that depend on initial conditions. Eventually, the state of the system undergoes a decoherence process when coupled to macroscopic measuring apparatus, so that its final state is described by the density matrix $\rho = p_{+\hat{\mathbf{n}}}|+\hat{\mathbf{n}}\rangle\langle+\hat{\mathbf{n}}| + p_{-\mathbf{n}}|-\hat{\mathbf{n}}\rangle\langle-\hat{\mathbf{n}}|$, with $p_{\pm\hat{\mathbf{n}}} = |\gamma_{\pm\hat{\mathbf{n}}}|^2$ denoting the probability for the measurement outcome of $|+\hat{\mathbf{n}}\rangle$ and $|-\hat{\mathbf{n}}\rangle$, respectively. This way, the field axis $\mathbf{B} = B\hat{\mathbf{n}}$ determines the possible outcomes of the spin-½ measurement process (either $|+\hat{\mathbf{n}}\rangle$ or $|-\hat{\mathbf{n}}\rangle$). Formally, we denote the operator describing the measurement process by $S_{+\hat{\mathbf{n}}} = |+\hat{\mathbf{n}}\rangle\langle+\hat{\mathbf{n}}| - |-\hat{\mathbf{n}}\rangle\langle-\hat{\mathbf{n}}|$. This operator is subjected to the well-known commutation relations: $[\mathbf{S_x},\mathbf{S_y}] = i\mathbf{S_z}$, $[\mathbf{S_y},\mathbf{S_z}] = i\mathbf{S_x}$, $[\mathbf{S_z},\mathbf{S_x}] = i\mathbf{S_y}$, as well as to the restriction on inferring simultaneously the components $\mathbf{S_x},\mathbf{S_y},\mathbf{S_z}$ of the spin (or equivalently the impossibility of inferring simultaneously all the components of $\mathbf{B}$).

Section 10: Measurement of the entire spin state without altering the spin

We have seen that under certain candidate theories of gravity, Bob can measure the axis of the spin without altering it. This capability seems to lead to a causality paradox, even without finding the entire state of the spin. In this section, we show a simple equivalence between measuring the spin axis and measuring the entire spin state. In other words, we show that any possibility to measure the spin axis would automatically enable to measure the spin direction, finding the only missing bit of information beyond the axis without altering the spin state. The direct implication is that finding the spin axis leads to a paradox that is equivalent to violation of the "no-cloning" theorem.

The idea in short is that once Bob finds the axis of the spin, even without its direction, he can place Stern-Gerlach magnets oriented along this axis, and thus find whether the spin is up or down without altering its state. Such a Stern-Gerlach test is only possible when Bob knows in advance what the axis is. This needed advanced knowledge is why finding the spin axis through the clocks leads to the paradox with no-cloning: Bob finds the entire spin state in two steps – finding the axis with clocks and the direction with Stern-Gerlach magnets.

We present this idea in more details with the following explanation. Any state of the form $\alpha|+z\rangle+\beta|-z\rangle$ can be expressed as a point on the Bloch sphere having the form $\cos\left(\theta/2\right)|+z\rangle+\exp(i\varphi)\sin\left(\theta/2\right)|-z\rangle$ and therefore, there always exists an axis $\hat{\mathbf{n}}$ to which the state is parallel $\alpha|+z\rangle+\beta|-z\rangle\propto|+\hat{\mathbf{n}}\rangle$, i.e., proportional up to a phase to $|+\hat{\mathbf{n}}\rangle$, where $n_x=\sin(\theta)\cos(\varphi),\ n_y=\sin(\theta)\sin(\varphi),\ n_z=\cos(\theta)$. Now, by symmetrically placing many clocks on a sphere Bob can find out that the axis of the spin up to a sign so he knows that it is either $|+\hat{\mathbf{n}}\rangle$ or $|-\hat{\mathbf{n}}\rangle$ (as in Appendix section 4). Finally, to find out the sign of the spin, Bob uses Stern Gerlach magnets so that the measurement axis will be parallel to the $\pm\hat{\mathbf{n}}$ axis. This way, the Stern-Gerlach measurement does not alter the spin state. By determining this last bit of information, whether the state of the spin is $|+\hat{\mathbf{n}}\rangle$ or $|-\hat{\mathbf{n}}\rangle$, Bob has the entire state of the spin (values of θ and φ as well as the values of $\beta / \alpha = \exp(i\varphi)\tan\left(\theta/2\right)$).

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